Key Misconceptions in Algebraic Problem Solving

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Abstract

The current study examines how holding misconceptions about key problem features affects students' ability to solve algebraic equations correctly and to learn correct procedures for problem solution. Algebra I students learning to solve simple equations using the Cognitive Tutor curriculum (Koedinger, Anderson, Hadley, & Mark, 1997) completed a pretest and posttest designed to evaluate their conceptual understanding of problem features (including the equals sign and negative signs) as well as their equation solving skill. Results indicate that students who begin the lesson with misconceptions about the meaning of the equals sign or negative signs solve fewer equations correctly at pretest, and also have difficulty learning how to solve them. However, improving their knowledge of those features over the course of the lesson increases their learning of correct procedures.

Keywords: algebraic problem solving; conceptual knowledge; learning; mathematics education

Introduction

Learners are not blank slates. Each student brings prior knowledge into a lesson, and that knowledge can greatly influence what the student will gain from the experience. Possession of good content knowledge has been associated with advantages in memory (Chi, 1978; Tenenbaum, Tehan, Stewart, & Christensen, 1999), generation of inferences (Chi, Hutchinson, & Robin, 1989), categorization (Chi, Feltovich, & Glaser, 1981), strategy use and acquisition (Gaultney, 1995; Alexander & Schwanenflugel, 1994), and reasoning skills (Gobbo & Chi, 1986; Johnson, Scott, & Mervis, 2004). It follows, then, that students who do not possess this important knowledge are at a disadvantage for successful problem solving or for learning new information.

In the domain of algebraic problem solving, one type of prior knowledge that is key to learning is conceptual understanding of features in the problem (e.g., equals sign, variables, like terms, negative signs, etc.). We operationally define conceptual knowledge of these features as not just recognizing the symbols or carrying out an operation, but understanding the function of the feature in the equation and how changing the location of the feature would affect the overall problem. In the next two sections, we describe why a lack of understanding about these features could interfere with students' *performance* and *learning* on procedural equation-solving problems.

Why conceptual knowledge of problem features should affect students' *performance*

Previous research has proposed that student misconceptions or gaps in conceptual knowledge of Algebra lead to use of incorrect, buggy procedures for solving problems (Anderson, 1989; Van Lehn & Jones, 1993). Use of incorrect procedures is common when learning Algebra (Lerch, 2004; Sebrechts, Enright, Bennett, & Martin, 1996), and by nature, this behavior inhibits accurate solution of problems.

One reason why use of these incorrect strategies may persist is that many of the procedures that students attempt to use are ones that will lead to a successful solution for some problem situations. Unfortunately, without adequate knowledge of the problem features, students are unable to distinguish between the situations in which the strategy will work and the ones where it is not applicable.

For example, one common strategy students have for solving equations is that if they want to remove a term from the equation, they subtract it from both sides of the equation. This works just fine for removing 4 from the equation $x + 4 = 13$. However, when they encounter equations like $x - 4 = 13$, many students still try to subtract 4 from both sides to solve the problem. One explanation for this mistake is that those students may be deficient in their conceptual knowledge of negative signs. If they don't process the fact that the negative sign modifies the 4 and is a necessary part of the "term" they are trying to remove, they ignore it, to the detriment of their goal of solving the problem.

Having good conceptual knowledge may thus be necessary for students to solve equations correctly, as deep strategy construction relies on inclusion of sufficient information about the problem features that make them appropriate or inappropriate. Unfortunately, for students with incorrect or incomplete conceptual knowledge about problem features, shallow strategies, such as the one described above, will likely prevail.

Why conceptual knowledge of problem features should affect students' *learning*

Instruction on procedural problems can take many forms, including demonstration by a teacher, study of written worked examples, hint messages from a computerized tutor, or feedback from a teacher or computer program. What all of these instructional techniques have in common is that in all cases, students need to be able align the presented information with the information in the problems they are or will be attempting to solve.

Carrying out this alignment is not trivial. Often times, people focus on literal similarity between the two sources -- which makes it difficult to identify the crucial features that have meaningful similarity -- as opposed to the overall structural similarity, which enables extraction of causal principles (Gentner, 1989)

Thus, students who do not have sufficient knowledge of problem features will likely only be capable of making shallow, surface analogies. If this activity leads to any learning at all, it can only be that of shallow procedures, which may be useful in some problem situations, but are not generally applicable.

In order to gain knowledge of deep procedures, students need to create deep analogies between the two sources of information. This requires noticing and attending to more than just surface features of the problem. Students need to have adequate conceptual understanding of the key features of the problem to fully grasp the meaning of instructional information. Thus, without this deep, meaningful knowledge of problem features, students will be unlikely to show great gains in procedural knowledge.

Which Features Matter?

Any given equation is comprised of a number of features that may be more or less crucial to its comprehension. These features include, but are not limited to, the equals sign (=), operation signs (+, -, ×, ÷), variables (*x*, *z*), and a variety of types of numbers (23, ¼, .65,) which may appear as constants, coefficients (e.g., .65x), exponents (e.g., x^{23}) or other roles in the equation. For this study, we used relatively simple linear equations, ones that can be solved in only a few steps (e.g., $3x + 5 = 9$, $12 = 4/x - 7$). Certainly, there are several features of these problems that are important for correct solution, but we focus on two features in particular: the equals sign and negative signs.

Understanding the equals sign has previously been shown to be crucial for algebraic problem solution (Knuth, Stephens, McNeil, & Alibali, 2006), and students' difficulty with the concept has been relatively well documented. Equality and the meaning of the equals sign in particular are difficult for students who are in the process of transitioning from arithmetic to algebraic thinking (Knuth et al., 2006), likely because students often think of the equals sign as an indicator of the result of operations being performed or the answer to the problem rather than of equivalence of two phrases (Baroody & Ginsburg, 1993). This kind of misconception greatly distorts perception of the equation and the intermediate goals necessary for its solution.

Another problem feature that seems important for solving early algebraic equations is the negative signs. Due to their abstract nature, working with negative numbers is inherently difficult for students who are transitioning from arithmetic to algebraic thinking (Linchevski & Williams, 1999). In algebra, students have to understand not only the magnitude, but the direction of numbers or terms in order to fully comprehend the problem (Moses, Kamii, Swap, & Howard, 1989). Failing to tie the negative sign to the term it modifies or to understand how changing or moving a negative sign impacts the equation should therefore preclude equation-solving success.

Thus, we expect that an incomplete or incorrect understanding of the role of the equals sign and negative signs will be detrimental to students' performance and learning of equation-solving procedures.

Measuring procedural performance and learning

There are two ways to measure students' procedural knowledge: the number of problems they are able to solve, and the number and type of incorrect procedures they use. These two measures may be linked in some sense, as use of incorrect procedures typically precludes correct problem solution. However, just because a student doesn't use an incorrect procedure doesn't mean that they know (and use) a correct procedure for solving the problem. Similarly, just because students decrease the number of incorrect procedures they use doesn't mean that they will learn to solve more problems correctly. Improvement in students' procedural knowledge requires both reduction in use of incorrect procedures and construction and strengthening of correct procedures (Siegler, 1996). Misconceptions about problem features likely influence both the number of related errors they make and their ability to construct a correct strategy that takes into account all of the important features in the problem. Thus, in this study, we examine both correct solutions and errors in order to evaluate students' procedural performance and learning.

Methods

Participants

Participating in this study were 49 high school students taking Algebra I with the Algebra Cognitive Tutor, a self-paced intelligent tutor system in which individuals use various representations (such as tables, symbols, and graphs) as they attempt to solve algebra problems (Koedinger et al., 1997). All students in the participating classes who had not yet begun the unit on solving two-step linear equations but had completed the prior problem-solving units took part in the study. Two students were excluded from analysis because they did not complete the posttest.

Measures

To assess students' procedural knowledge, we used 8 experimenter-designed items that measured ability to effectively carry out procedures to solve problems. These items were representative of the types of problems taught in Algebra I courses, but were designed to be slightly more challenging than the majority of problems taught in the particular Tutor unit by either frequently including the features of interest (e.g., negative signs) or adding features the students had not encountered before (e.g., two variable terms).

To assess students' conceptual knowledge, we used 40 experimenter-designed items that measured understanding of concepts that seemed crucial for success in Algebra. In this paper, we focus on two such concepts: the equals sign (7 items) and negative signs (10 items).

Finally, as a measure of students' general math ability, we utilized two algebra-related released items from the Trends in Mathematics and Science Study (TIMSS; Mullis et al., 2003). Examples of the experimenter-designed items can be found in Figure 1.

Procedural items:

$$-4x + 7 = 5$$

$$9 = \frac{-6}{b}$$

Conceptual items:

State whether each of the following is a like term for 6c:

a.	3d	Yes No
b.	-4c	Yes No
c.	-5	Yes No
d.	8c	Yes No
e.	5(c-1)	Yes No
f.	6	Yes No

State whether each of the following is equal to 3 – 4x:

a.	3 + 4x	Yes No
b.	3 + (-4x)	Yes No
c.	4x – 3	Yes No
d.	-4x + 3	Yes No
e.	4x + 3	Yes No

Figure 1: Example conceptual and procedural assessment items.

Procedure

Participating students were administered a paper-and-pencil test assessing their conceptual and procedural knowledge of algebra; there were two forms of the test and half of the students were randomly assigned to receive each. Following completion of the pretest, students began the two-step linear equations unit with the Tutor. During the unit, the Tutor provided guided procedural practice solving two-step equations; students received immediate feedback about any errors, and could ask the Tutor for hints if they were unsure of what to do. While they worked through the unit, log data of interactions with the Tutor were collected for each student. After each student completed the unit, he or she was administered the alternate version of the paper-and-pencil test as a posttest.

Results

Performance on Procedural Test Items

Answers to the procedural items were coded as correct or incorrect, and we computed the percent of problems answered correctly by each student at pretest and at posttest. Procedural problems were also coded in terms of the error (if any) that was made (e.g., combined non-like terms, used the wrong operation, deleted a negative, etc.), and composite scores were created to indicate the number of conceptual errors of each type (negative signs and equals sign) students made on the problems.

Students solved an average of 32% of procedural problems correctly on the pretest and 33% on the posttest.

Performance on Conceptual Test Items

Conceptual items were coded as correct or incorrect, and we computed the percent of problems answered correctly by each student at pretest and posttest. Students answered an average of 59% of the conceptual items correctly on the pretest and 63% on the posttest.

In addition, we computed the percent of correct responses for subsets of the conceptual problems (e.g., those that target knowledge of the equals sign or of negative signs) to determine the amount and quality of students' knowledge about those particular concepts. Students answered an average of 58% of the equals sign items correct at pretest and 57% at posttest; pretest and posttest scores for negative sign items were 57% and 61%, respectively.

Effect of Conceptual Knowledge on Procedural *Performance*

To examine how conceptual knowledge of the equals sign and negative signs influences correct equation-solving performance, we correlated students' percent of equals sign and negative signs-related conceptual items answered correctly with their percent of equations solved correctly. Greater conceptual knowledge of the equals sign ($R(47) = .52$, $p<.01$) and negative signs ($R(47) = .48$, $p<.01$) were both positively correlated with the number of correct solutions.

To determine whether students' lack of conceptual knowledge led to use of certain incorrect procedures for solving equations, we correlated knowledge of the equals sign with equality-related errors (e.g., performing an operation to only one side of an equation, dropping the equals sign from the equation, etc.) and knowledge of negative signs with negatives-related errors (e.g., deleting negatives, subtracting terms that should be added, etc.). Students with lower knowledge of equals signs made more equality errors ($R(47)= -.30$, $p<.05$); a similar nonsignificant trend was found for negative sign knowledge ($R(47) = -.26$, $p<.10$).

Effect of Conceptual Knowledge on Procedural *Learning*

There are two ways to investigate the role of conceptual knowledge in procedural learning. The first is whether lack of conceptual knowledge at pretest hinders learning, as it is more difficult to decipher instruction, thus leading to weak gains in procedural knowledge. A second possibility is that students don't have to have the conceptual knowledge from the start, but that if they make gains in it over the course of the lesson, similar gains in procedural knowledge will follow.

In order to test the contribution of either or both of these factors, we conducted a pair of regression analyses to predict procedural learning, separate for each concept. For each concept, we first entered pretest percent of equations solved (to control for amount of improvement possible) plus pretest conceptual knowledge of that concept and then gain in knowledge of that concept. We then conducted the analysis in the opposite way, first entering pretest procedural performance plus gain in conceptual knowledge, followed by pretest conceptual knowledge.

As shown in Table 1, for both the equals sign and negative signs, gain in conceptual knowledge predicted improvement on the procedural problems beyond that predicted by pretest conceptual knowledge. However, for both concepts, a trend was found showing that pretest conceptual knowledge tended to also be a useful predictor of learning: students who came in with lower amounts of conceptual knowledge at pretest learned less over the course of the lesson.

One alternate possibility is that conceptual knowledge is not itself crucial to learning, that better students simply learn more from instruction about both concepts and procedures. To rule out this possibility, we conducted a similar pair of regression analyses, entering students' performance on the TIMSS items as well as pretest procedural performance before pretest conceptual knowledge and improvement in conceptual knowledge. For both features, prior conceptual knowledge (equals sign $p < .05$, negatives $p < .10$) and improvement in conceptual knowledge (equals sign $p < .01$, negatives $p < .05$) were meaningful predictors of procedural learning above and beyond that predicted by more general math ability.

Unpacking the Potential Learning Event

Case Study Illustration: When Learning Occurs At pretest, Student JU answered only 30% of the negative signs-related conceptual items correctly. Among his incorrect answers on these items were endorsing that -4x + 3 was the same as both 4x + 3 and 4x – 3, and indicating that the negative sign in -9b was not part of the term. These answers are consistent with the misconception that negatives can enter and exit phrases without consequence and that their locations (and connections to numbers or variables in the problem) are not significant.

When solving the 8 equations on the pretest, he made a total of 5 negative sign errors, such as leaving negatives out of the equation when moving on to the next step or attempting to subtract in order to remove a term which is already negative; these types of errors comprised 63% of the total number of errors he made. He solved only 25% of the equations correctly on the pretest.

When working with the Tutor, he was given the problem y/-8 + (-6) = -6. His first step to solve this problem was to try to subtract 6 from both sides. He received a bug message from the Tutor, stating that "Since -6 is negative, you should add to remove it from the left side. Erase your last step and add 6 to both sides." He went on to solve the problem correctly. Several problems later, he encountered a similar equation: -4 = y/5 + (-2). This time, he immediately added 2 to both sides.

On the posttest, he answered 50% of the negative signs-related conceptual items correctly; the main improvement was that he now included negative signs as parts of the terms they modify. He made only 2 negative sign errors when solving equations (33% of the total number of errors he made). He solved 50% of the equations correctly on the

Table 1. Pretest knowledge and gain in knowledge of the equals sign and negative signs as predictors of improvement in percent of equations solved correctly

Analysis	$R^2_{initial}$[1]	$R^2_{pre + gain}$	Added R^2	Significance
Equals sign				
1. Pretest conceptual knowledge entered before gain in conceptual knowledge	.05	.27	.22	$p < .01$
2. Gain in conceptual knowledge entered before pretest conceptual knowledge	.21	.27	.06	$p < .10$
Negative signs				
1. Pretest conceptual knowledge entered before gain in conceptual knowledge	.05	.17	.12	$p < .05$
2. Gain in conceptual knowledge entered before pretest conceptual knowledge	.10	.17	.07	$p < .10$

Note: [1] In all cases, $R^2_{initial}$ includes pretest performance on the procedural items (to control for the amount of improvement possible) plus the first factor entered.

posttest; 25% (or 2 equations) more than on the pretest. Thus, he showed improvement both on his conceptual understanding of negative signs and his ability to solve equations correctly.

Case Study Illustration: When Learning Doesn't Occur
At pretest, Student NR answered none of the equals sign-related conceptual items correctly. Among his incorrect answers on these items was his assertion that the equals sign "means that the number that is being added, subtracted, multiplied, or divided. It's what it equals." This answer is consistent with the misconception that the purpose of the equals sign is to show the answer to the problem rather than to show that the expressions on either side are equivalent.

When solving the 8 equations on the pretest, he made a total of 8 equals sign errors, such as performing operations to only one side of the equation and trying to put all of the terms on one side of the equals sign; these types of errors comprised 44% of the total number of errors he made. He solved only 13% of the equations correctly on the pretest.

When working with the Tutor, he was given the problem $2x + (-9) = -3$. He first attempted to subtract 3 from both sides, trying (incorrectly) to get all terms before the equals sign. When the tutor indicated that it was incorrect, he attempted to subtract 2 from both sides. After realizing that he can simplify the signs to get to $2x - 9 = -3$, he then tries to subtract 3 from both sides again. He then received a bug message, telling him to "Focus on the side of the equation with the variable. The variable is on the left side." He tries again to subtract -3 from both sides, and upon receiving the same bug message again, tries to add -3. Finally, he gives up and goes to the other side of the equation to move the -9.

On the posttest, he still failed to answer any of the equals sign-related conceptual items correctly and still indicated that the equals sign "means the answer." Some improvement was shown, in that he made 3 equals sign errors when solving equations (33% of the total number of errors he made). However, he still only solved 13% of the equations correctly on the posttest. Thus, he did not improve his conceptual knowledge of the equals sign or his ability to solve equations correctly.

Discussion

Results suggest that having incorrect or incomplete pretest conceptual knowledge of the equals sign and negative signs is associated with use of related, incorrect strategies for solving algebraic equations; these students have difficulty solving equations correctly. In addition, students who hold these misconceptions at pretest tend to learn less from instruction on how to solve equations; this relation exists even when more general math ability is controlled for. However, having good conceptual knowledge at pretest may not be entirely crucial, as improving this knowledge over the course of instruction does increase the amount of learning that students achieve; the two case studies highlight the differences between what can happen when a student gains conceptual knowledge and when they do not.

The current study focused on the equals sign and negative signs as the key features necessary for learning to solve simple algebraic equations. Previous research has also shown that misconceptions about what constitutes like terms causes similar problems when students attempt to solve (and learn to solve) more difficult problems (Booth, Koedinger, & Siegler, 2007). Thus, at various points in the learning process, misconceptions or gaps in conceptual knowledge of relevant features inhibit students' performance and learning. But the good news is that even if they don't begin a lesson with this crucial knowledge, as long as they get it along the way, they can close some of the gap. These results suggest that providing students with these conceptual prerequisites should be an important goal for Algebra courses, and perhaps the math courses that lead up to it.

Current instructional methods, including those used in the Cognitive Tutor, are not typically focused on helping students gain conceptual understanding even though the need for greater emphasis on conceptual understanding has been acknowledged (NCTM, 2000, National Mathematics Advisory Panel, 2007). The IES Cognition Practice Guide has recommended several instructional methods that could help increase students' conceptual understanding, including studying worked examples and self-explaining solutions (Pashler et al., 2007). Preliminary findings in the current line of research indicate that incorporating a combination of these methods with procedural practice in the Tutor can improve both students' conceptual and procedural knowledge of algebra (Booth, Siegler, & Koedinger, in press); further research is necessary to determine when and for whom these techniques are useful.

Conceptual knowledge is presumed to increase learning in algebra because it helps students to draw deep analogies between instruction and practice problems; however, this hypothesis was not tested in the current study. Future research will need to assess students' techniques while working with the problems to determine whether they are indeed drawing these analogies and the quality thereof. Another outstanding question is precisely why increased conceptual knowledge yields increased procedural learning in algebra. There are at least two possible mechanisms that could explain the finding. First, it could be that students learn the concept, which helps them to understand the lesson better, which leads to greater learning of correct procedures; this view is consistent with the iterative model of conceptual and procedural development (Rittle-Johnson, Siegler, & Alibali, 2001). An alternate possibility is that students with greater conceptual knowledge are able to solve more equations correctly (presumably because they notice and attend to the appropriate features in the problem and apply correct strategies), so perhaps gains in conceptual knowledge just helped students to notice and attend to the correct features and apply correct strategies they already knew, but weren't using. We hypothesize that it is the former, but further research is needed to tease apart these possibilities.

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References

Alexander, J.M., & Schwanenflugel, P.J. (1994). Strategy regulation: The role of intelligence, metacognitive attributions, and knowledge base. *Developmental Psychology, 30,* 709-723.

Anderson, J.R. (1989). The analogical origins of errors in problem solving. In D. Klahr & K. Kotovsky (Eds). *Complex information processing: The impact of Herbert A. Simon.* Hillsdale, NJ, England: Lawrence Erlbaum Associates, Inc.

Baroody, A., & Ginsburg, H. (1983). The effects of instruction on children's understanding of the "equals" sign. *Elementary School Journal, 84*, 199–212.

Booth, J.L., Koedinger, K.R., & Siegler, R.S. (2007). [Abstract]. The effect of prior conceptual knowledge on procedural performance and learning in algebra. In D.S. McNamara & J.G. Trafton (Eds.), *Proceedings of the 29th Annual Cognitive Science Society.* Austin, TX: Cognitive Science Society.

Booth, J.L., Koedinger, K.R., & Siegler, R.S. (in press). Using self-explanation to improve algebra learning. *Proceedings of the 30th Annual Cognitive Science Society.*

Chi, M.T.H. (1978). Knowledge structures and memory development. In R.S. Siegler (Ed.), *Children's thinking: What develops?* (pp. 73-96). Hillsdale, NJ: Erlbaum.

Chi, M.T.H., Feltovich, P.J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive Science, 5,* 121-152.

Chi, M.T.H, Hutchinson, J.E., & Robin, A.F. (1989). How inferences about novel domain-related concepts can be constrained by structured knowledge. *Merrill-Palmer Quarterly, 35,* 27-62.

Gaultney, J.F. (1995). The effect of prior knowledge and metacognition on the acquisition of a reading comprehension strategy. *Journal of Experimental Child Psychology, 59,* 142-163.

Gentner, D. (1989). The mechanisms of analogical learning. In S. Vosniadou & A. Ortony (Eds.), *Similarity and analogical reasoning* (pp. 199-241). London: Cambridge University Press. (Reprinted in Knowledge acquisition and learning, 1993, 673-694).

Gobbo, C., & Chi, M. (1986). How knowledge is structured and used by expert and novice children. *Cognitive Development, 1,* 221-237.

Johnson, K.E., Scott, P., & Mervis, C.B. (2004). What are theories for? Concept use throughout the continuum of dinosaur expertise. *Journal of Experimental Child Psychology, 87,* 171-200.

Knuth, E. J., Stephens, A. C., McNeil, N. M. & Alibali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education, 37*, 297-312.

Koedinger, K.R., Anderson, J.R., Hadley, W.H., & Mark, M.A. (1997). Intelligent tutoring goes to school in the big city. *International Journal of Artificial Intelligence in Education, 8*, 30-43.

Lerch, C. M. (2004). Control decisions and personal beliefs: Their effect on solving mathematical problems. *Journal of Mathematical Behavior, 23*, 21-36.

Linchevski, L., & Williams, J. (1999). Using intuition from everyday life in 'filling' the gap in children's extension of their number concept to include the negative numbers. *Educational Studies in Mathematics,* 39, 131 – 147.

Moses, R., Kamii, M., Swap, S.M., & Howard, J. (1989). The Algebra Project: Organizing in the spirit of Ella. *Harvard Educational Review 59(4)*, 423-443.

Mullis, I. V. S. et al. (2003). *TIMMS Assessment Framework and Specifications 2003 (2nd Edition)*. Boston, Massachusetts: International Study Center.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics: Higher standards for our students . . . higher standards for ourselves.* Washington, DC: Author.

National Mathematics Advisory Panel (2007, January). Preliminary Report.

Pashler, H., Bain, P., Bottge, B., Graesser, A., Koedinger, K., McDaniel, M., and Metcalfe, J. (2007). *Organizing Instruction and Study to Improve Student Learning* (NCER 2007-2004). Washington, DC: National Center for Education Research, Institute of Education Sciences, U.S. Department of Education.

Rittle-Johnson, Siegler, and Alibali. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93, 346-362.*

Sebrechts, M.M., Enright, M., Bennett, R.E., & Martin, K. (1996). Using algebra word problems to assess quantitative ability: Attributes, strategies, and errors. *Cognition and Instruction, 14*, 343.

Siegler, R.S. (1996). Emerging minds: The process of change in children's thinking. New York: Oxford University Press.

Tenenbaum, G., Tehan, G., Stewart, G., & Christensen, S. (1999). Recalling a floor routine: The effects of skill and age on memory for order. *Applied Cognitive Psychology, 13,* 101-123.

Van Lehn, K., & Jones, R.M. (1993). What mediates the self-explanation effect? Knowledge gaps, schemas, or analogies? In M. Polson (Ed.) *Proceedings of the fifteenth Annual Conference of the Cognitive Science Society* (pp. 1034-1039).