Making Informed Decisions in Educational Technology Design:
Toward Meta-Cognitive Support in a Cognitive Tutor for Geometry

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Objectives
We have been pursuing an approach to educational technology called "Cognitive Tutors." Cognitive Tutors combine cognitive science research, artificial intelligence techniques, and the wisdom of practicing teachers to provide instructional assistance in the context of rich problem solving activities. We have developed complete high school mathematics courses that integrate Cognitive Tutors, text materials, and teacher practices. In large-scale evaluations of our Algebra Cognitive Tutor course (Koedinger, Anderson, Hadley, & Mark, 1997) and more recently our Geometry Cognitive Tutor course, we have documented significant student achievement gains over control classes in both higher-level problem solving and basic skills. A critical research and development goal of both projects has been codifying student knowledge, at multiple levels of acquisition, in computer-based cognitive models built within the ACT-R theory (Anderson & Lebiere, 1998, Koedinger & MacLaren, 1997). We have worked closely with our teacher partners in local urban schools, where students are having particular difficulties with mathematics, to regularly assess and improve our course materials and Cognitive Tutors. The Geometry project added two research goals: 1) to model and support meta-cognitive as well as cognitive skill acquisition and 2) to better elaborate our development methodology. In this paper, we provide a slice of our design history that will provide specific examples of progress toward these goals.

A development question of critical importance to educational software designers is: How to best decide between design alternatives in the face of limited development resources? Ideally, we would implement the alternatives and evaluate their relative impact on student learning. However, such experimentation is not possible for all design decisions, especially given the high costs of programming and controlled experimentation. Seeking the advice of practicing teachers helps, but often teachers will suggest “do both.” Theory and prior research can be good guides, but often such sources are either not specific enough or in conflict. In this paper, we will show how we used “Difficulty Factors Assessment” as an empirical guide for making informed design decisions that are most likely to benefit student learning.

The PACT Geometry Cognitive Tutor
Our Geometry Cognitive Tutor consists of six major units addressing the following topics: Area, Pythagorean Theorem, Angles, Similar Triangles, Quadrilaterals, and Circles. The software learning environment includes verbal and visual presentation of problem scenarios, a set of representational tools including a spreadsheet and equation solving tool, and feedback and messaging windows for tutorial assistance. The underlying artificial intelligence technology makes it possible for Cognitive Tutors to provide instructional assistance in the context of more open-ended problem solving activities that allow for multiple strategies and solution paths. Such individualized solution-sensitive support stands in contrast to the feedback on single-step multiple choice or fill-in the blank questions that is typical of commercially available Computer Aided Instruction (CAI) systems and more recent web-delivered systems.

PACT Geometry is different from past Cognitive Tutors in that we have tried to facilitate the learning not only of cognitive skills, like computing the area of a figure, but of meta-cognitive skills, like recognizing a knowledge gap and searching for information to fill it. When students are having difficulty with a problem-solving step, instead of hinting at the relevant knowledge and how to apply it (a cognitive-level hinting strategy), the tutor hints toward a meta-cognitive “look-it-

up” strategy. It suggests searching the on-line glossary for relevant knowledge and, if necessary, helps students in finding and applying a relevant geometric principle. In this paper we will focus on meta-cognitive strategies for finding, understanding, and correctly applying Area formulas. In a previous experiment with the Angles unit, we demonstrated how the tutor could support a meta-cognitive strategy of explaining problem-solving steps and how that helped students acquire deeper, more flexible knowledge (Aleven, Koedinger, & Cross, 1999).

Assessment Data Supports a Design Idea: Tutoring Flexible Use of Formulas

This paper describes slices of the PACT Geometry design history related to the initial Area unit of the tutor - similar methods were applied in the design of the other units. In our initial designs of the Area unit, we drew upon past cognitive research in algebra. This research identified critical differences distinguishing more routine knowledge application, of the kind often targeted in traditional CAI, from deeper, more flexible knowledge use. In particular, it distinguished simpler single-step ‘decomposed’ problems from harder multi-step ‘composed’ problems (Heffernan & Koedinger, 1997). It also distinguished forward applications of knowledge from backward applications (Koedinger & MacLaren, 1997). We hypothesized that these factors would apply in geometry as well, and designed activities and tutorial assistance to achieve more flexible knowledge use. For instance, the tutor helps students learn to solve composed problems by supporting them in breaking a complex problem down into simpler problems (Polya, 1957). This support comes in the form of encouraging students to add columns in their spreadsheet to store intermediate results on the way to a final goal. For example, if a student needs help in finding the scrap metal left when a can lid is cut from a square piece of metal, the tutor will hint: “The area of the scrap metal is equal to the area of the square minus the area of the lid. You can add 2 columns from the table menu to compute the area of the square and the area of the lid first.”

Assessments of PACT Geometry support these original design decisions. We designed a difficulty factors assessment (DFA) for Area, and pre- and post-tested students to assess their learning successes and difficulties. We administered these tests in our participating classrooms at three urban high schools over the four years of our development effort. The data we report here is from the 1998-99 school year. The DFA results confirmed that backward application of Area formulas (e.g. finding the height of a parallelogram from its area and base) is significantly more difficult than forward application and that composed problems are significantly more difficult than single-step problems. Looking at the instructional impact of the tutor from pre-test to post-test, we saw that students improved on all problem types, forward (30% to 67%), backward (18% to 49%), and composed (3% to 39%). We suspect that if the tutor had only focused on single-step forward problems, as many commercial geometry CAI programs do, students would not have acquired flexible knowledge that would transfer (Singley & Anderson, 1989) to more difficult backward and composed problems.

Assessment Data Rejects a Design Idea: Tutoring Diagram-Drawing

The original tutor was not designed to support students in drawing diagrams for problems – each problem on the tutor included a diagram. Because diagram drawing is thought to be an important skill to learn (Polya, 1957), we designed a DFA manipulation to test whether this would be a useful skill to target in the tutor. Three alternative versions of each problem were distributed across multiple test forms: 1) diagram provided, 2) no diagram provided, or 3) a written hint that drawing a diagram would help to solve the problem. We found only small, statistically non-significant differences at the pre-test between providing a diagram (20%), hint (18%) and no diagram (14%). By the post-test, students improved in all categories (to 58%, 48%, and 50% respectively) despite no explicit instruction on diagram drawing within the tutor. Using these post-test results, we can estimate students diagram drawing abilities at 86% proficiency (diagram drawing (86%) times solving with diagram (58%) equals solving without diagram (50%)).

Students performed just about as well when they had to draw a diagram on their own as when they

were provided one. Given the reality of limited development resources, we decided that student learning would be better served by spending effort on development goals other than creating an interface and tutorial assistance for diagram drawing. Enhancing our learning environment with diagram-drawing support remains a long-term goal, but it is now a lower priority.

**Assessment Data and Teachers Drive Redesign: Applying Formulas with Understanding**

Following our initial design of the Area unit, we used DFAs to drive further redesign. We designed some problems on the assessment to tap another dimension of flexible knowledge use besides backward application and composition. We were interested in distinguishing shallow application of knowledge from deeper knowledge use that is resilient to irrelevant information and incorrect perceptual over-generalization. Specifically, we created parallelogram area problems where the givens not only included the length of the base and height, but also the length of the other, non-base, side of the parallelogram. If students find the area of a rectangular as the product of two adjacent sides of the rectangle (rather than as the product of the base and height), they may over-generalize this approach to parallelograms where it produces an incorrect result. We found evidence that students do so. We compared student performance on this “parallelogram-with-distractor” problem to a trapezoid area problem that did not have a distractor. Because the trapezoid formula \((A = \frac{1}{2}h \times (b_1+b_2))\) is more complicated, we would expect it to be more difficult than a typical parallelogram problem. However, we found the parallelogram-with-distractor problem to be more difficult than the trapezoid problem (36% correct vs. 42% correct). In further error analysis, we found a large proportion of student errors on this problem involved the over-generalization of multiplying the sides rather than the base and height.

In addition to such quantitative analysis, our redesigns were also influenced by the observations of teachers using the tutor. In one focus group meeting, teachers observed that an earlier version of the tutor was too formula-driven, rather than concept-driven – helping students apply formulas, but not doing enough to help them understand when and why formulas are correct. These discussions provided supporting qualitative evidence that the tutor needed to better engage students in deeper processing of the principles underlying formulas and their application.

To further students’ ability to apply formulas with understanding, we added a new row in the spreadsheet requiring students to fill in the names of segments playing particular roles in the formula. A key part of the process of applying the parallelogram formula with understanding is to correctly identify the base and height of the parallelogram. Many problems do not require such understanding because they either directly provide the values of the base and height, or because only two numbers are given and students simply use them without knowing why it is appropriate to do so. By adding the “Geo Name” row to the table, we made explicit the previously implicit step of identifying the base and height. Thus, the tutor is able to provide students with feedback and assistance on this step. If a student incorrectly identifies one of these segments, a message appears explaining that the base of a parallelogram and its respective height must be perpendicular, and why the entered label does not fit this criterion (Figure 1).

To further target the ability to identify what segments are necessary in applying a formula, we added problems that used non-standard orientations of height and base. We also updated our on-line glossary to better support the reasoning behind Area formulas. We included examples to illustrate the concept underlying the formula, as well as its application (Figure 2).

**Conclusion**

We have used data from “Difficulty Factors Assessments” and teacher focus groups to make informed design decisions in the development of the PACT Geometry Cognitive Tutor. Our goal has been to prioritize our development efforts on improvements that are most likely to enhance student learning. The general strategy is to differentiate knowledge elements (skills, strategies, meta-cognitive skills) that are relatively easy for students to learn from knowledge elements that

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remain difficult to learn. In some cases, like the example of diagram drawing skills, DFA results indicated that current instruction may be adequate and development efforts would be better directed elsewhere. In other cases, like the example of shallow formula application, DFA results identified weaknesses in the current design and guided a redesign. By the time of the AERA meeting, we hope to present new data on the impact of these latest redesigns.

References


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**Figure 1.** Part of the PACT Geometry Tutor illustrating the addition of the "Geo Name" row in the spreadsheet. The goal is to make explicit the otherwise implicit step of identifying the relevant role fillers, like base and height, in a formula.
The area $A$ of a parallelogram is equal to the base ($b$) multiplied by the height ($h$).

CAUTION: The height of a parallelogram is not necessarily equal to any of its sides!

**Example:**

In the parallelogram $ABCD$, the base ($AB$) = 6 ft, the side ($AD$) = 5 ft, and the height ($BE$) = 3 ft.

The parallelogram can be made into a rectangle, so you can see why the area ($A$) of the parallelogram:

$$A = AB \times BE = 6 \times 3$$

$$= 18 \text{ square ft.}$$

**Figure 2.** Concept-based definitions and examples.