COMPARING INSTRUCTIONAL STRATEGIES FOR INTEGRATING CONCEPTUAL AND PROCEDURAL KNOWLEDGE

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We compared alternative instructional strategies for integrating knowledge of decimal place value and regrouping concepts with procedures for adding and subtracting decimals. The first condition was based on recent research suggesting that conceptual and procedural knowledge develop in an iterative, hand over hand fashion. In this iterative condition, conceptual and procedural lessons were interleaved. The second condition followed the common ordering of conceptual lessons before procedural lesson (concepts-first condition). All lessons were presented via a computer-based intelligent tutoring system and seventy-two sixth-grade students participated. Students in the iterative condition made greater improvements in procedural knowledge and comparable improvements in conceptual knowledge, compared to the concepts-first condition. Students in both groups did better when problems were presented in money contexts rather than symbolically. Both the iterative ordering of lessons and presenting problems in money contexts reduced students' digit alignment errors when adding and subtracting decimals.

Introduction

Competence in mathematics rests on children developing and linking their knowledge of concepts and procedures (Bisanz & LeFevre, 1992; Hiebert, 1986; Hiebert & Wearne, 1996; Silver, 1986; Sowder, 1992). However, competing theories have been proposed regarding the developmental relations and instructional importance of conceptual and procedural knowledge. Whether concepts or procedures develop first has been hotly debated within psychology (e.g., Gelman & Williams, 1998; Siegler, 1991; Siegler & Crowley, 1994; Sophian, 1997). Similarly, the "math wars" for how we should improve mathematics instruction have pitted conceptual understanding against procedural skill as the most important goal of mathematics instruction (Mathematically Correct, 2000; National Council of Teachers of Mathematics [NCTM], 2000). The mathematics education research community has begun to move beyond this dichotomy and recognize the importance of each type of knowledge. However, *how* conceptual and procedural knowledge are related is still not well understood. We propose that conceptual and procedural knowledge influence one another in mutually supportive ways and build in an iterative process. The purpose of this study is to compare instruction on decimals that is based on these mutually supportive, iterative relations to instruction based on the common sequence of concepts before procedures.

Developing Conceptual and Procedural Knowledge

Recent research indicates that conceptual and procedural knowledge often develop in an iterative process, with improvements in one type of knowledge leading to improvements in the other type of knowledge, which leads to further improvements in the first type of knowledge (Rittle-Johnson, Siegler & Alibali, 2001). For example, limited understanding of some domain concepts guides students' attention to important problem features and facilitates generation and use of a correct procedure. In turn, developing this procedural knowledge leads to improvements in conceptual understanding, perhaps by freeing cognitive resources for noticing patterns and relations, highlighting the importance of certain problem features, or revealing certain misconceptions.

Applying this iterative process to instruction could aid learning for several reasons. A "big idea" in reforming mathematics instruction is developing students' conceptual understanding of mathematics (NCTM, 2000). Although the general wisdom of developing students' conceptual understanding in a domain early in instruction is intuitive and an improvement over old practices, there are several reasons why iterating between conceptual and procedural instruction could lead to even greater learning.

First, presenting all the conceptual material first could overwhelm students' working memory and lead to confusion. Rather, some level of procedural fluency reduces working memory load (e.g., Shrager & Siegler, 1998) and could facilitate further improvements in conceptual knowledge. Second, iterating between conceptual and procedural lessons could help highlight the relevance of each lesson type for the other, avoiding the common problem of students not integrating conceptual and procedural instruction (e.g., Resnick & Omanson, 1987). Finally, iterating between varied, but related, tasks could support appropriate generalization of concepts and procedures, and thus reduce overgeneralizations (applying a concept or procedure in an inappropriate way) and undergeneralizations (failing to transfer to appropriate tasks) (Anderson, 1993).

Although this iterative instructional approach appears promising, past research does raise several cautions. First, conceptual understanding can be used to generate and choose good procedures (e.g., Geary, 1994; Gelman & Williams, 1998; Hiebert & Wearne, 1996), so it may be important to begin procedural instruction after students have firm conceptual understanding. Furthermore, some studies have found that prior procedural instruction can interfere with learning concepts (e.g., Warrington & Kamii, 1998). Thus, it is important to compare the effects of an iterative ordering of conceptual and procedural instruction to sequential ordering of conceptual instruction before procedural instruction (see Table 1).

Using Context to Elicit and Build Upon Informal Knowledge

An additional big idea in mathematics instruction is eliciting students' prior, informal knowledge through real-world contexts (NCTM, 2000). Real-life story problems can be easier for students to solve than matched symbolic problems, partially because the context helps students avoid common errors (Carraher, Carraher, & Schliemann, 1985; Koedinger & Nathan, 2000; Rittle-Johnson & Koedinger, 2001). Nevertheless, national assessments indicate that U.S. students do poorly on story problems and often do worse on story problems than on symbolic computation problems (Baranes, Perry, & Stigler, 1989; Lindquist, 1989). It is clear that not all story contexts elicit students' informal knowledge. Furthermore, there is little guidance for how to integrate context into conceptual and procedural instruction. Context could facilitate learning of particular types of knowledge. For example, context could facilitate links between conceptual and procedural knowledge because the grounded, informal context makes it easier to see the relations between the two types of knowledge. In this case, introducing procedural instruction in context immediately after conceptual instruction using the same context could facilitate links between the two types of knowledge. Context could also help students to bridge from grounded, informal concepts to abstract, mathematical concepts. This would suggest presenting instruction on formal mathematical concepts immediately after contextualized conceptual instruction. The two instructional conditions in this study contrast these two potential benefits of presenting problems in context (see Table 1).

Table 1. Lesson Orders for the Two Conditions

Concepts-first condition	*Iterative (context-first) condition*
1. Contextualized concept lesson	1. Contextualized concept lesson
2. First abstract concept lesson	2. Contextualized procedure lesson
3. Second abstract concept lesson	3. First abstract concept lessons
4. Contextualized procedure lesson	4. First abstract procedure lesson
5. First abstract procedure lesson	5. Second abstract concept lesson

In summary, we compared alternative instructional strategies for integrating knowledge of decimal place value and regrouping concepts and procedures for adding and subtracting decimals. We evaluated whether iterating between instruction on decimal concepts and procedures, initially within money contexts, led to greater learning than instruction that covered decimal concepts in context and in the abstract before lessons on decimal procedures. We also evaluated whether money contexts elicited informal knowledge and supported better performance on a decimal assessment. We predicted that iterating between lessons on decimal concepts and procedures would lead to greater learning and that providing money contexts would improve performance.

Method

Participants

The intervention was a component of a sixth-grade mathematics curriculum that we are developing. Four classes of sixth-grade students at our two pilot schools participated. Each class period was assigned to one of the two conditions. Eighty-three students began this study and 72 completed the intervention and assessments, with an equal number of students in the two conditions.

Intervention

The current study focuses on our computer-based intelligent tutoring systems for decimal concepts and procedures. The Cognitive Tutors provide on-demand, step-specific help at any point in the problem-solving process and immediate feedback on errors (Koedinger, Anderson, Hadley, & Mark, 1997). We designed three conceptual lessons on decimal place value and regrouping and three procedural lessons on adding and subtracting decimals.

In the conceptual lessons, students were asked to enter a number in a place value chart and then to show the value of the number in novel ways using regrouping (e.g., 6.0 as 6 ones, as 5 ones and 10 tenths, etc.; see Figure 1). In the first conceptual lesson, the problems were presented in a money context and using money terminology for the place values (e.g., dimes, pennies). On later conceptual lessons, the problem format was the same, but no context was given and symbolic place value names were used (e.g., tenths, hundredths).

In the procedural lesson, students were given word problems that required adding or subtracting two decimal numbers (see Figure 2). Students entered the numbers in a chart and completed the computations. In the first procedural lesson, problems were

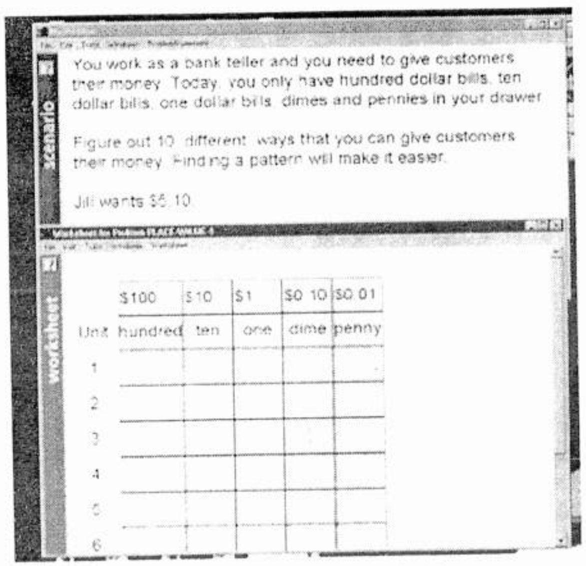

Figure 1. Screen shot of a problem in the contextualized conceptual lesson.

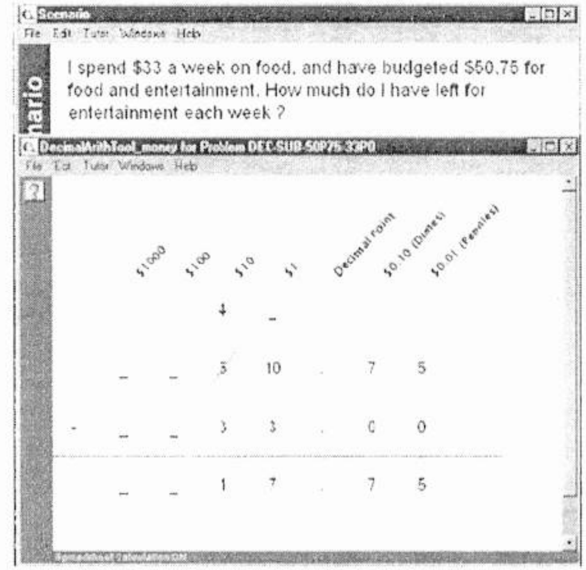

Figure 2. Screen shot of a problem in the contextualized procedural lesson.

presented in a money context, and monetary place value column labels were included on the chart. In the second lesson, problems were in non-money (often unfamiliar) contexts and the standard place value labels were included on the chart. In the third lesson, the chart did not include place value labels.

The order of these lessons varied by condition. Both orders began with the conceptual lesson on place value and regrouping in a money context. In the *concepts-first* condition, the other two conceptual lessons were presented next. Then, the three procedural lessons were presented, beginning with the lesson in context. In the *iterative* condition, the second lesson was the procedural lesson in a money context. Then, students completed the second conceptual lesson, followed by the second procedural lesson, and so forth (see Table 1).

Students worked on the Cognitive Tutor twice a week for four to eight weeks, with an average time of 4 hours and 10 minutes spent on the decimal lessons.

Assessment

The pretest and posttest followed a 2 (Knowledge type: conceptual or procedural) x 2(Context: money or none) x 2 (Learning type: learning or transfer) design, with 1 learning and 2 transfer problems in each of the other cells, for a total of 12 questions. See Table 2 for an example of each question type. We generated two versions of the

Table 2. Example Assessment Items

	Conceptual		Procedural	
	Learning	Transfer	Learning	Transfer
Money Context	Show 5 different ways that you can give Ben $4.07.	2 dimes are worth how many pennies?	You had $8.72. Your grandmother gave you $25 for your birthday. How much money do you have now?	You buy a super-size candy bar for $1.12, a bag of chips for $3.39 and a pack of soda for $4. What is your total cost?
No Context	List 5 different ways to show the amount 4.07	2 tenths are worth how many hundredths?	Add: 8.72 +25	Add: 1.12 + 3.39 + 4

assessment so that the same problem was presented in a money context and with no context (kind of "Difficulty Factors Assessment" see Koedinger & Nathan, submitted).

Results

An alpha value of 0.05 was set as the criteria for all statistical analyses.

Pretest

We conducted a repeated-measures ANOVA on percent correct at pretest, with knowledge type, context and learning type as within-subject factors. Students solved more problems correctly when they were presented in a money context, rather than without context (63% vs. 31% correct), $F(1,82) = 114.18$. As shown in Figure 3, the benefits of context were greater on conceptual knowledge items than on procedural knowledge items, $F(1,82) = 5.25$. Comparisons of performance on individual items suggested that the money contexts reduced alignment errors when adding and subtracting (3% vs. 18% of relevant problems) and elicited knowledge of the relations between different place values, such as the number of pennies in a dime, on the conceptual items. Finally, students did better on the procedural knowledge items than on the conceptual knowledge items (59% vs. 44% correct), $F(1, 82) = 9.96$, and on the transfer items than the learning items (56% vs. 44% correct), $F(1,82) = 13.44$, though in both these comparisons other factors were not controlled and may account for the differences.

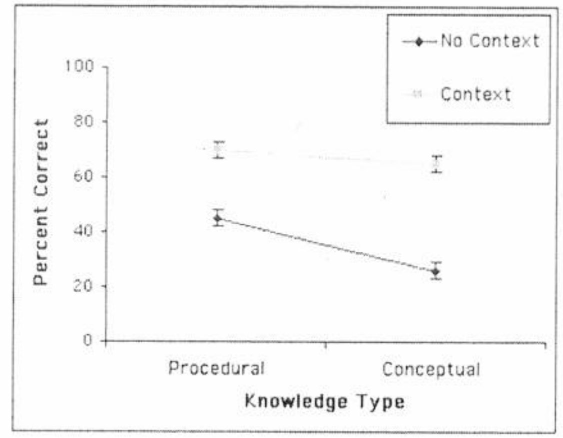

Figure 3. Effect of context on conceptual and procedural items at pretest.

Effects of Condition on Learning

For each student, we calculated his or her gain score as the percent correct at posttest minus percent correct at pretest. To evaluate the effects of condition on learning, we conducted a repeated-measures ANOVA on the gain scores, with condition as a between-subject factor and knowledge type and learning type as within-subject factors. There was a main effect for condition, $F(1, 72) = 3.92$. As shown in Figure 2, students in the iterative condition made greater gains than students in the concepts-first condition (23 vs. 13 percentage point gain). There was also an interaction between condition and knowledge type, $F(1, 72) = 4.91$, such that the effect of condition was mostly on the procedural knowledge items. The effect of condition did not differ between the learning and transfer items. Comparisons of performance on individual items suggested that the iterative ordering was most beneficial on items where the "align numbers on the right" pre-conception seemed most difficult to override - when a two-digit whole number is added to a two-digit decimal number (e.g. 8.72 + 25). Students in the iterative condition made a 25% gain on this item, compared to no gain on this item in the concepts-first condition. Furthermore, students in the iterative condition only made alignment errors on 6% of relevant arithmetic problems at posttest, compared to 14% in the concepts-first condition.

Why did the iterative condition lead to better learning of decimal arithmetic procedures? Informal observations and quantitative data from the intervention suggested that the iterative ordering highlighted the links between the conceptual and procedural lessons. The conceptual task of representing a number in many different ways was novel to the students and very challenging. To solve a single conceptual problem during the intervention, students spent 6 to 7 minutes, made an average of 6 errors and 1 or 2 help requests. The procedural task of adding and subtracting decimals was a familiar task for sixth graders and was much easier for them (2 to 3 minutes per problem, and less than 1 error and 1 help request per problem). Informal observations suggested that recognizing that ideas of borrowing from decimal subtraction could be applied to the conceptual task of representing a number in multiple ways facilitated understanding and performance on the conceptual task. If this was the case, students in the iterative condition should have done better on the later conceptual lessons, which were presented after a procedural lesson for these students (see Table 2). Although not a statistically reliable difference, students in the iterative condition outperformed students in the concepts-first condition on the second and third conceptual lessons. Students in the iterative condition took less time per problem (7 min vs. 8.2 min), made fewer errors (6.2 vs. 8.3 errors per problem) and asked for less help (2.1 vs. 2.7 help requests per problem) than students in the concepts-first condition (recall that during the intervention, students worked on a problem until it was correct). Students in the two conditions had similar performance on the second and third procedural lessons. Linking to students' knowledge of borrowing during the conceptual lesson may have strengthened this knowledge and helped to build connections between conceptual and procedural knowledge.

Discussion

Both presenting decimal problems in money contexts and iterating between conceptual and procedural lessons on decimals led to greater success. First, these results contradict the common belief that word problems are harder than symbolic problems (Koedinger & Nathan, 2000). Students solved more problems correctly when they

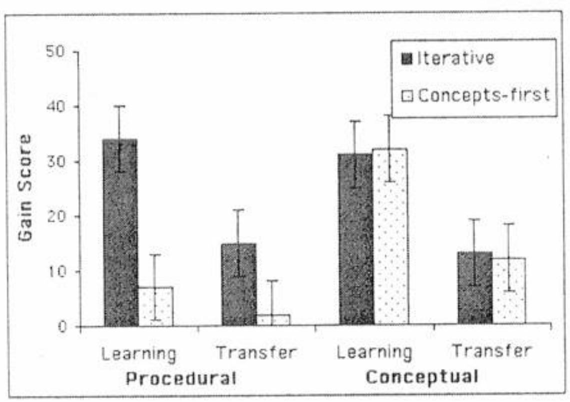

Figure 4. Effect of condition on gain scores on the conceptual and procedural assessments.

were presented in money contexts, possibly because it tapped students' informal knowledge of needing to align digits of the same place value before adding and subtracting and of the relations between different place values for re-grouping.

Our findings also suggest that reform efforts to develop students' conceptual knowledge before their procedural knowledge need to be refined. Iterating between conceptual and procedural knowledge lessons, first in money contexts and later without money contexts, led to greater learning than presenting all the conceptual lessons first. This iterative sequencing was particularly beneficial for procedural knowledge. It seemed to help students avoid an over-generalization of the "align digits on the right" procedure that works with whole numbers. Introducing the procedural task early, and interleaving it with conceptual instruction, seemed to help link and strengthen knowledge shared by the conceptual task and the procedural task. These findings support an iterative model for the development of conceptual and procedural knowledge and help move beyond the debate on which type of knowledge develops first or is more important.

These findings also illustrate the interplay between research and practice. Theoretical constructs and laboratory research on the development of conceptual and procedural knowledge and on the effects of context guided the design of our classroom-based instructional intervention. This intervention led to improved student learning, and the classroom data provided empirical evidence to constrain theories on learning and teaching conceptual and procedural knowledge.

References

Anderson, J. R. (1993). *Rules of the mind.* Hillsdale, NJ: Erlbaum.

Baranes, R., Perry, M., & Stigler, J. W. (1989). Activation of real-world knowledge in the solution of word problems. *Cognition and Instruction, 6*, 287-318.

Bisanz, J., & LeFevre, J.-A. (1992). Understanding elementary mathematics. In J. I. D. Campbell (Ed.), *The nature and origins of mathematical skill* (pp. 113-136). Amsterdam: Elsevier Science.

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1985). Mathematics in the streets and in the schools. *British Journal of Developmental Psychology, 3*, 21-29.

Geary, D. C. (1994). *Children's mathematical development: Research and practical applications.* Washington, DC: American Psychological Association.

Gelman, R., & Williams, E. M. (1998). Enabling constraints for cognitive development and learning: Domain specificity and epigenesis. In D. Kuhn & R. S. Siegler (Eds.), *Handbook of child psychology: Cognition, perception, and language* (5th ed., Vol. 2, pp. 575-630). New York: John Wiley.

Hiebert, J. (1986). *Conceptual and procedural knowledge: The case of mathematics.* Hillsdale, NJ: Erlbaum.

Hiebert, J., & Wearne, D. (1996). Instruction, understanding and skill in multidigit addition and instruction. *Cognition and Instruction, 14*, 251-283.

Koedinger, K. R., Anderson, J. R., Hadley, W. H., & Mark, M. A. (1997). Intelligent tutoring goes to school in the big city. *International Journal of Artificial Intelligence in Education, 8*, 30-43.

Koedinger, K. R., & Nathan, M. J. (2000). Teachers' and researchers' beliefs of early algebra development. *Journal of Mathematics Education Research, 31*, 168-190.

Koedinger, K. R., & Nathan, M. J. (submitted). *The real story behind story problems. Effects of representations on quantitative reasoning*. Manuscript submitted for peer review.

Lindquist, M. M. (1989). *Results from the Fourth Mathematics Assessment of the National Assessment of Educational Progress*. Reston, VA: NCTM.

Mathematically Correct. (2002). Retrieved from http:://mathematicallycorrect.com

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Resnick, L. B., & Omanson, S. F. (1987). Learning to understand arithmetic. In R. Glaser (Ed.), *Advances in instructional psychology* (Vol. 3, pp. 41-95). Hillsdale, NJ: Erlbaum.

Rittle-Johnson, B., & Koedinger, K. (2001). *Concepts, procedures & context: Difficulty factors for learning fraction arithmetic.* Paper presented at the Biennial Meeting of the Society for Research in Child Development, Minneapolis, MN.

Rittle-Johnson, B., Siegler, R.S., & Alibali, M.W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93*, 346-362.

Shrager, J., & Siegler, R. S. (1998). A model of children's strategy choices and strategy discoveries. *Psychological Science, 9*, 405-410.

Siegler, R. S. (1991). In young children's counting, procedures precede principles. *Educational Psychology Review, 3*(2), 127-135.

Siegler, R. S., & Crowley, K. (1994). Constraints on learning in nonprivileged domains. *Cognitive Psychology, 27*, 194-226.

Silver, E. A. (1986). Using conceptual and procedural knowledge: A focus on relationships. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics*. Hillsdale, NJ: Erlbaum.

Sophian, C. (1997). Beyond competence: The significance of performance for conceptual development. *Cognitive Development, 12*, 281-303.

Sowder. (1992). Making sense of numbers in school mathematics. In G. Leinhardt, R. Putman, & R. Hattrup (Eds.), *Analysis of arithmetic for mathematics teaching* (pp. 1-51). Hillsdale, NJ: Erlbaum.

Warrington, M. A., & Kamii, C. (1998). Multiplication with fractions: A Piagetian, constructivist approach. *Mathematics Teaching in the Middle School, 3*, 339-343.