TWO STRATEGIES ARE BETTER THAN ONE: MULTIPLE STRATEGY USE IN WORD PROBLEM SOLVING

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Much research on skill and skill acquisition in math and science has focused on the formal representations and procedures that are the stuff of traditional textbooks (Anderson, Greeno, Kline, & Neves, 1981; Larkin, McDermott, Simon, Simon, 1980). However, a number of more recent concerns have shifted their focus to informal representations and strategies involved in perceptual intuition (Larkin & Simon, 1987; Koedinger & Anderson, 1990; Tabachneck, 1992), situated cognition (e.g., Greeno, 1989) or other types out-of-school learning (e.g., Collins, et. al., in press). There is little question among educational researchers and practitioners that formal procedures and representations are difficult for students to learn. Furthermore, they are often not used outside of school, and those people who do effectively solve mathematical problems in everyday situations often employ common sense strategies that are different than those taught in school (e.g., Carraher, Carraher, & Schlieman, 1987; Lave, 1984; Resnick, 1987; Rogoff, 1984; Scribner, 1984). Although some might intend it to, the trend toward legitimatization of informal strategies should not be read as a claim that they are the mainstay of skill and that the formal representations and procedures of traditional schooling are unnecessary. Instead, we agree with the position that it is through the <u>integration</u> of common sense and school-taught knowledge that the greatest understanding of mathematics is achieved (Carpenter & Fennema, 1992; Cobb et al., 1991; Janvier, 1987; Kaplan, Yamamoto, & Ginsburg, 1989; Kaput, 1985, 1989; Koedinger & Anderson, 1991; Lampert, 1990; Lesh, Post, & Behr, 1987; Lester, 1980; Nathan, Kintsch, & Young, 1992; Schoenfeld, 1989). This paper provides verbal protocol evidence in the domain of mathematical word problem solving that the integrated use of formal and informal strategies is more effective than the strict use of a single strategy. We provide a theoretical interpretation of why this is so and are working on a computer simulation of this theory.

In looking in detail at the strategies of college students solving simple algebra word problems, we observed a high frequency of three unschooled, non-algebra strategies. In contrast to both opposing positions on the importance of unschooled strategies, we found that *no single strategy*, schooled or unschooled, is significantly more effective than any other. Instead, the key finding was that the use of <u>multiple strategies</u> within a single problem is significantly correlated with success. We analyzed the strengths and weaknesses of each strategy and claim that the unschooled strategies' strengths lay mainly in the processes of comprehension and translation, and the schooled strategies' strength lay mainly in the calculation process. Thus, schooled and unschooled strategies can complement each other in the process of solving a single problem.

We are developing a cognitive model of multiple strategy use in the ACT-R production system to better understand how and why multiple strategy use provides an advantage. Computer modeling forces a "discipline of detail" that requires a characterization of multiple strategy use that is precise and unambiguous. In addition, computer models provide a sufficiency

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* The order of authors is arbitrary and was decided by a thumb wrestling context. We thank Mitchell Nathan for his comments. This research has been supported by MDR-92-53161 from the National Science Foundation and by the McDonnell Foundation.

Paper presented at the 1994 Annual Meeting of the American Educational Research Association, New Orleans, LA, April 4-8.

proof that each strategy can produce correct solutions. By inspecting the resulting models, we can more directly see the unique features and strengths of the various strategies.

METHOD

Twelve Carnegie Mellon undergraduates (6 male and 6 female) participated in the study for class credit. Subjects were simply asked to solve "the practice problems"; algebra was never mentioned Subjects were trained to give a "think-aloud" concurrent verbal protocol (c.f., Ericsson and Simon, 1984); they were audio-taped. The task consisted of two two-part algebra word problems:

Problem 1: A man has 3 times as many quarters as he has dimes. The value of the quarters is one dollar and 30 cents more than the value of the dimes.
Q1a: How many dimes does the man have? (answ: 2)
Q1b. If the total value of the coins had been 2.55, how many coins would the man have had altogether? (answ: 12)
Problem 2: A board was sawed into two pieces. One piece was two-thirds as long as the whole board; it exceeded the second piece in length by 4 feet.
Q2a: How long was the board before it was cut? (answ.: 12 feet)
Q2b: How long is the second piece? (answ.: 4 feet)

These problems might be criticized as traditional word problems unlike those emphasized by current reforms (e.g., NCTM, 1989; NAEP, 1993). However, one of the key criticisms of these problems, that they lead to the routine application of over-learned prototypical schemas, does not apply in this case. Students had great difficulty with these problems indicating that they were novel for them. We observed a variety of non-routine problem solving efforts which is consistent with the kind of thinking being emphasized by current reforms. Traditional problems have also been criticized for their lack of relevance to real world problem solving and for motivational reasons. However, the processes identified here are quite likely to generalize to more authentic problems. Establishing this is a goal of our current research.

The task was presented as a Hypercard stack on a Macintosh IIx computer. The answers to problem 2b were very brief since most subjects had calculated the answer in the previous problem, hence problem 2b was dropped from the analyses. Thus, we analyzed 36 solution protocols (12 subjects, 3 problems each).

Protocol analysis method

The protocols were classified into five stages relating to problem solving:

1. Parsing and Understanding - reading or rereading of problem statement and question, and superficial transforms of the problem statements.
2. Solution Setup - setting up the problem solution, e.g., formulating an equation.
3. Solution - carrying out the computations associated with the setup.
4. Solution Answer - finding the answer resulting from the previous computation
5. Problem Answer - finding the answer to the problem question using the solution answer.

The analysis revealed a principled episode segmentation: each time subjects went to a stage listed prior to the current behavior, a new episode was begun. The vast majority of the episodes began with a re-situation in the problem, i.e., with re-reading, closely re-phrasing, or simple transforms of the problem statements (stage 1 behavior). The episode would wind its way serially through the next four stages to a solution, or would abort somewhere on the way.

Subjects were identified as using 4 different kinds of strategies. Each involved particular movement between representations , or translations, and manipulations within a representation, or transformations (cf. Lesh, Post, & Behr, 1987) as illustrated in Figure 1.

In a complete coding of the 36 protocols we identified the following four strategies:

1. Algebra (ALG): The most formal strategy employed by subjects. The verbal problem statement is translated to algebraic assignments and equations. The equations are transformed to find a solution (solve for the unknown).
2. Model-based reasoning, heuristic (MH): The verbal problem statement is translated into arithmetic constraints, represented either verbally or as written arithmetic. A value for an unknown is guessed at and that value is propagated through the constraints. Resulting values are checked against given values to determine whether or not a contradiction is reached. If so, further guesses are be made until a consistent value is found.
3. Verbal-math (VM): The verbal problem statement is transformed into alternative verbal forms. There are two types of transformations: 1) verbal recodings intended to facilitate translation or 2) qualitative operations to estimate unknown values. Included in this strategy are translations to "verbal algebra" where equations are described verbally and transformations are performed that are analogous to written algebra transformations.[1]
4. Diagrammatic (DG): The verbal problem statement is translated into a diagrammatic representation. Transformations are performed on the diagram, including annotations and diagram supported inferences.

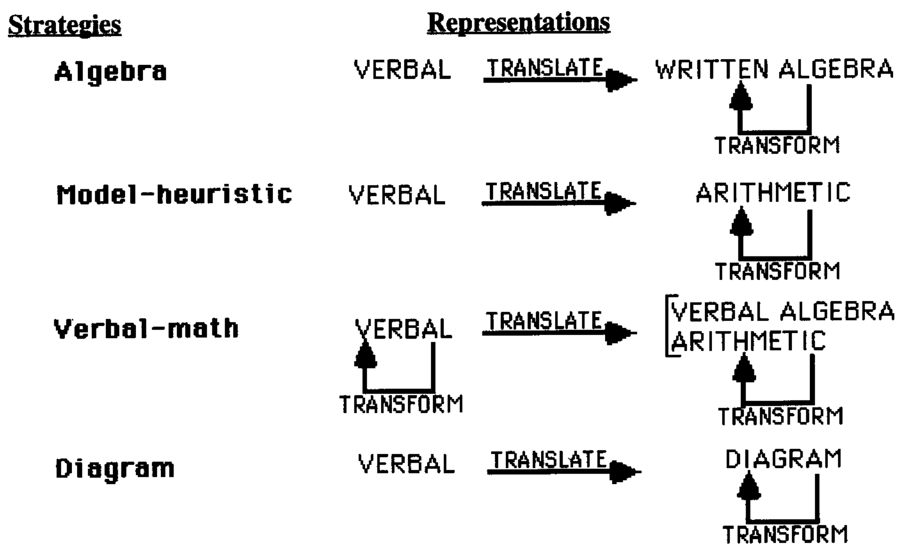

Figure 1. Representations involved in the four strategies.

Because Verbal-math strategy is perhaps the least familiar, we provide an example of it below. The example is from problem 1a, S13, final episode: Note that the "two dimes" in line 22 are not a guess as in the MH strategy, but rather, derived or estimated through verbal reasoning.

18 Ok, Each one is 25 cents ...
19 How can you have a dollar 30 in quarters?
20 That would be 5 quarters.
21 It's *a dollar 30 more in quarters.*
22 So, it's got to have at least 2 dimes
23 that'll be 6 quarters ...
24 and 6 quarters is 3 times as many as 2 dimes ...
25 so ...
26 he has 2 dimes, hopefully

[1] Verbal algebra was performed by subjects in one condition of Mayer (1982) and is a generalizations of the "ratio" strategy identified by Hall, et. al. (1989).

QUANTITATIVE RESULTS & DISCUSSION

The key result of this study was that students were more effective when they used multiple strategies in solving a problem than when they stuck with (or got stuck with) a single strategy. Of the 19 solutions in which more than one strategy was used, 15 of them or 79% were correct. All 19 solutions involved at least one switch from a schooled to an unschooled strategy (or vice versa). Of the 17 solutions involving a single strategy, 7 of them or 41% were correct (X2(36,1) = 4.2, p = .02). In other words, multiple strategy use was about twice as effective as single strategy use. Other candidate features of successful performance (algebra use, algebra effectiveness, other strategy use and/or effectiveness, interactions with aptitude and time spent) did not distinguish problem solving success from failure. For example, subjects were neither more nor less effective when using algebra than they were when using other strategies. 54% (7/13) final uses of algebra led to a correct solution while 65% (15/23) final uses of other strategies led to a correct solution — a difference which was not statistically significant.

To get a clearer understanding of this "multiple strategy" effect, we examined subjects' problem difficulties or impasses. See Figure 2. On some solutions attempts (36%) subjects did not reach an impasse, either because a correct solution was found without trouble or because an error went unnoticed (54% successes). When solution attempts involved an impasse, reapplying the same strategy, or giving up in one case, always led to failure (0% successes), while switching strategies was far more likely to be successful (79% successes). It is important to note that it is not persistence that matters here – in general, failed solution attempts tend to take as long or longer than successes (see the bottom row of Figure 2) – rather it is having an alternative strategy to fall-back on that is critical.

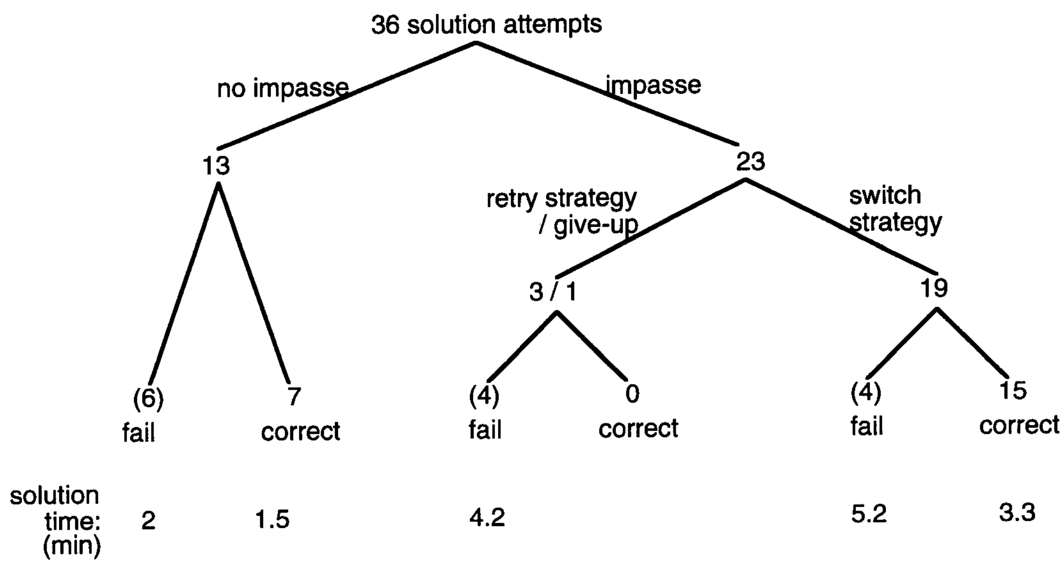

Figure 2. Impasses and Strategy Switching

TOWARD A COGNITIVE MODEL OF MULTIPLE STRATEGY USE

Our modeling effort focuses on clarifying the role of sense-making during problem solving. We model how subjects' informal strategies operate and how the use of these strategies in concert with algebra circumvents impasses and leads to greater problem-solving performance than single strategy use. In particular, the current model illustrates cognitive processes behind these major phenomena:

- *How conceptual errors occur* -- these can be generated by over-general knowledge (e.g., merging quantity and some intensive measure of quantity) or superficial application of knowledge (e.g., incorrectly inferring logical structure from the syntactic structures).
- *How sense-making strategies can avoid errors.*

The model is written within the ACT-R theory which has a long history of success as a unified theory of cognition (Anderson, 1983, 1990, 1993). ACT-R is more appropriate than competing cognitive theories in its ability to account for symbolic problem solving as well as more continuous and subtle processes like strategy selection.

Analyzing the strengths and weaknesses of alternative strategies

Scope of the multiple strategy effect

Multiple strategies will not be effective in routine or easy tasks, as problem solvers can apply a single special-purpose strategy with high success. Novel tasks with a 50% chance of a correct solution may be optimal. Further, in non-routine tasks, multiple strategies will only be more effective than a single strategy if errors made with the one strategy are not the same errors made with another strategy, that is, if multiple strategies are somewhat independent in their probability of being effective. This will happen only if they have features which complement each other.

Differences in strategy features

Aside from representational differences, features on which to compare strategies include: 1) prior familiarity to students and the familiarity of the representation in which they are performed (e.g., model-heuristic is performed within the familiar representations of words and arithmetic, though the strategy itself may or may not be familiar), 2) computational efficiency , and 3) the demands the strategies put on working memory capacity. Our task analysis identified the strengths and weaknesses of each strategy for these problems as described in table 1.

TABLE 1: Strategy differences

	FAMILIARITY	COMPUTATIONAL EFFICIENCY	CAPACITY DEMANDS
ALG	low/medium	high	medium
MH	high	low	low
VM	high	varies	high
DG	medium	varies	low

Applying the model to illustrate strategy differences

Broadly, word problem solving involves three subtasks: 1) comprehension, 2) translation and 3) calculation. We provide an analysis of the effects of strategy differences on these subtasks in terms of the processes illustrated in Figure 1 and the dimensions shown in Table 1.

Algebra: The emphasis is on correct translation. Often, comprehension is carried only so far as it is necessary to obtain algebraic structures. Such a process resulted in one fairly consistent bug, "value-number", in our example problem: *A man has 3 times as many quarters as he has dimes.* is translated to "quarters is 3 x, dimes is x", whereupon *The value of the quarters is one dollar and 30 cents more than the value of the dimes.* is translated to 3x-x=130. Note that by assigning 3x to the number of quarters and x to the number of dimes, one strips the units (and the associated meaning) from the abstract representation. This error can be construed as a form of over-generalization – merging a given quantity with some intensive measure of quantity.

Abstraction is exactly what makes algebra so general and efficient; but this efficiency comes at a cost: Algebra does little to maintain or support validation of the problem solver's understanding of the problem situation. Capacity demands are higher for the comprehension/translation phases, but lower for calculation since everything is written down uniformly.

Model example: Appendix 1 shows the verbal protocol of subject 3 on problem 1a, who makes the value-number bug and repeats the algebra strategy three times. The student does not find the bug and settles for the wrong answer. Alongside the verbal protocol is an abstracted trace of production firings in the model. This solution attempt is one of the four in the retry strategy/give-up category in Figure 2.

Model Heuristic: This guess-and-test strategy keeps most of the problem statement structure intact, thus few to no translation problems occur. The strategy aids comprehension, since by reifying the problem with numbers many times in subsequent guesses, subjects become very familiar with the content. There are low capacity demands: subjects can remain in the familiar verbal representation, use the available written problem statements, and only need to keep track of a small amount of arithmetic, which can be written down. The value-number bug did not appear in either the MH or in the VM strategy in any of the protocols. Because these strategies remain in the verbal representation, the meanings of quantities and their role in the problem (e.g., the number of dimes) are maintained. Unit clashes, as in "three quarters minus one dime should equal a dollar thirty", are quite salient.

Model example: Appendix 2 shows the verbal protocol of subject 9 on problem 1a, who begins with the algebra strategy and makes a similar error to S3. This strategy is then abandoned in favor of MH. Remaining within a verbal representation makes the over-generalization shown earlier apparent to this subject, so the bug is now corrected. The model trace begins the same as S3's; it is not included here. We show a trace of the MH strategy. This solution attempt was coded as a strategy switch in Figure 2.

Verbal-Math: This is the only strategy that has transformations on the verbal representation before any translation is done (see Figure 1). In the comprehension task, subjects try to transform the statements into a solution-enabling format, where translation into a math representation is facilitated. This task is capacity demanding: such transformation are rarely written down and must thus be remembered in their entirety, while enough capacity must be left to transform them. Its computational efficiency varies, since problem statements vary widely in how easily they can be reduced to a solution enabling form. If the comprehension task is successful, translation generally only consists of putting numbers into the verbal representation. Verbal math too is a familiar strategy, since it is mostly played out in the well-known verbal representation.

Diagram: Like in the algebraic strategy, the comprehension task consists of attempts to translate the problem statements into a very different representation. However, this representation results in a structure that maintains some of the semantics of the problem. For instance, in solving problem 1a one subject drew rows of bigger circles (quarters) and smaller circles (dimes). It is this semantic-preserving quality that makes it easy to understand. Translation and calculation demand little capacity since the structures are drawn on paper in sequential small steps. The computational efficiency varies: problems don't lend themselves equally well for diagramming, and it is not always easy to draw the diagram in such a way that the unknown is easily computable (or even visible).

Summary. Clearly, individual strategies have inherent weaknesses. Our model illustrates how switching representations can make problem solving more successful by compensating for these weaknesses. Some unschooled strategies (e.g., VM and MH) naturally support comprehension by retaining the problem semantics, thus avoiding common conceptual errors. In the multiple strategy example provided, the strategies were used non-interactively. Other protocols show how strategies can be used interactively as results from an intermediate strategy feed back into the original one. For example, subjects may start with the algebra strategy, run into an error, turn to another strategy to aid the comprehension and translation subtasks, and go back to algebra to solve the problem. Multiple strategy use allows one to adapt to the local difficulties of a problem through selection of a strategy that is best for each subprocess.

CONCLUSION

Problem solvers regularly use unschooled strategies. Since integrating such common sense strategies and formal mathematics has met with instructional success (Carpenter & Fennema, 1992; Cobb et al., 1991), exactly how these strategies function and interact with previously learned problem-solving strategies is a matter of great interest to cognitive scientists and educators. We have begun to provide a detailed explanation, in the form of a running cognitive model, of how multiple strategy use helps solvers to understand a problem and to compensate for the weaknesses of a given strategy and their own limitations in executing it. Our intention is to have a complete model of the cognitive processes involved in selecting, performing, and integrating multiple strategy use in algebra problem solving.

The emphasis here is not on the identification of strategic alternatives to algebra (though the VM strategy for verbal algebra is somewhat novel) -- others have identified similar strategies (e.g., Baranes et al, 1989; Hall et al., 1989). The emphasis is on understanding how these strategies are used interactively, one strategy providing a sense-making function when another failed. To understand this compensatory interaction, we have identified dimensions of strengths and weaknesses of these strategies: representational format, familiarity, computational efficiency, and capacity demands. A major focus is to understand, through modeling, the specific circumstances under which each strategy is effective both as a basic research goal and for pedagogical purposes. Further work will identify issues in strategy development intended to provide a theoretical explanation of how more familiar student-developed representations and strategies (e.g., guess-and-check, verbal algebra) can be used as ramp in the acquisition of unfamiliar formal representations and strategies (written algebra) (Kaput, 1989).

Formal strategies facilitate computation because of their abstract and syntactic nature, however, the abstraction process can lead to nonsensical interpretations and syntactic errors. The remedy to preventing or repairing such errors is not more careful reapplication of the formal strategy, rather problem solvers are better off switching to an informal strategy to make sense of the problem situation or to identify conceptually inconsistent slips. We explained the multiple strategy effect in terms of the complementary relationship between the computational efficiency of formal strategies and the sense-making function of informal strategies. Effective problem solving in novel situations results from opportunistic and flexible application of both formal and informal strategies.

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APPENDIX 1: DATA-MODEL MATCH

1a	**Verbal report,Subject 3, Problem 1a**		**Model output of buggy algebra version, Problem 1a**
line:	Features: Value-of error, impasse & strategy retries	cycle:	Features: Value-of error, impasse & strategy retries
1-3	[Reads problem statement]	1	Ah, an algebra problem.
4	So, we're going to say dimes are equal to x -	2-6	X = DIMES 3X = QUARTERS
5	number of dimes is equal to x	7	[Reading 'value-of DIMES' as 'DIMES']
6	and the number of quarters is 3x	8	[Reading 'value-of QUARTERS' as 'QUARTERS']
7 - 9	[repeats 3-6]		
10	and the value of the quarters is one dollar and 30 cents more than the value of the dimes	7-9	QUARTERS is DIMES plus ONE-DOLLAR-AND-30-CENTS.
		11	They want to know the number of DIMES.
11	so, 3x = x + 130	12-13	(3 X) = X + 130
12	so then I subtract x from the one side,		
13	I get 2x=130		
14	x= 2/130 which is - uh		
15	6 - 65 cents,	14-15	X = 65
16	which does not make sense.	16	Solution value 65 does not make sense. DIMES should be an integer
17- 33	[Repeats algebra and error twice more]		[Note: Like the subject, the model will continue to retry algebra and make the same error.]
34	I get x = 65.		
35	So I have a value - it's not making sense, though.		

APPENDIX 2: DATA-MODEL MATCH

1b	**Verbal report,Subject 9, Problem 1a**		**Model output of buggy-algebra + model-based version, Problem 1a**
line:	Features: Value-of error, impasse & strategy switch. Non-interactive use of strategies.	cycle:	Features: Value-of error, impasse & strategy switch. Non-interactive use of strategies.
1-14	[Buggy algebra translation. Similar to panel 14.]	1-16	[Buggy algebra translation. Same as panel 15.]
15-16	[Rereads problem indicating impasse and switches to the Model-based strategy]	17-18	I'm going to try the guess-and-test method.
		19-28	[Recomprehending problem sentences.] QUARTERS is DIMES plus ONE-DOLLAR-AND-30-CENTS.
17	right- so if he had - if he had 2 dimes,	29	Guessing value for DIMES is 2.
18	that would be 20 cents,	30	2 DIMES would be 0.2 DOLLARS
19	and if he had a dollar 50 in quarters,		
20	how many would that be?		
21	That would be - well, if he had two dimes, that would be 20 cents.	31	Because DIMES-IN-DOLLARS is 0.2 DOLLARS and ONE-DOLLAR-AND-30-CENTS is 1.3 DOLLARS, QUARTERS-IN-DOLLARS is 1.5 DOLLARS.
22	and then, 20 + 130 would be 150.		1.5 DOLLARS would be 6.0 QUARTERS
23	and that would be - how many quarters would that be? ... that would be 4 - 5 -6		
24	that would be 6 quarters	32-35	Because DIMES is 2 and 3 is 3, QUARTERS is 6.0. That's right! That's what I got before.
25	2 times 3 is 6 - oh, ok. Easy.		Now what were they asking for?
		36	The problem was asking for DIMES.
26	Uh - the man has 2 dimes.		DIMES was found to be 2.