Student learning of negative number: A classroom study and difficulty factors assessment

Kathryn A. Verzoni, West Point School
Kenneth R. Koedinger, Carnegie Mellon University

Purpose

This study illustrates a productive collaboration of a practicing teacher-researcher (first author) and a cognitive researcher (second author) exploring the important subtleties in middle school students' development of basic and application knowledge of negative numbers. The study combines two experiments: 1) a "difficulty factors assessment" given before and after instruction to tap critical hurdles in negative number learning, and 2) an experimental instructional approach involving student construction of computer models intended to contextualize and make more meaningful negative number concepts. This "life-situated modeling" approach was developed by the teacher-researcher and follows from her prior success in applying this approach to other areas of middle school math (Verzoni, 1996). The difficulty factors assessment was developed in collaboration with the cognitive researcher and follows from previous research addressed at identifying key trends in cognitive strategy development in mathematics (Koedinger & Tabachneck, 1995; Nathan, Koedinger, & Tabachneck, 1996).

The difficulty factors assessment investigated how the following factors affect performance of sixth, seventh and eighth graders on problems involving negative numbers: 1) problem presentation (story vs. number sentence), 2) number of arithmetic operations (one vs. two), 3) position of the unknown (start vs. result), and 4) for the number-sentence problems only, representation of the unknown (box vs. blank). This study also investigated the short term effects of a sixth grade instructional intervention that involved learners in the creation of contextualized computer simulations of integer operations. Specifically, the purpose of this pseudo-experimental pre-post design was to assess the effectiveness of the instructional intervention for developing skill with negative number operations. Do interactive, constructive, highly contextualized learning experiences involving computer simulated representations of negative number operations contribute to gain in ability to solve negative number problems?

Theoretical Framework

The current algebra reform movement (Kaput, 1995, p.5) stresses the importance of building upon children's language learning abilities and their natural generalizing and abstracting powers in order to deepen and extend their reasoning about quantity, number, space, and uncertainty. Instead, in algebra education we generally emphasize procedures for symbol manipulation and solving for the unknown. Our habits have been associated with weaknesses in students' abilities to connect algebraic representations with life-situated relationships (McCoy, 1994; Monk, 1989; Wollman, 1983). These inabilities may be behind the difficulties that the American citizenship in general has with relating high school learned algebra to everyday problem situations.

Issues regarding beneficial aspects of contextualized problems are of particular importance to this study. Carraher, Carraher, and Schliemann (1987) found enhanced problem solving performances among children when problem contexts evoked "street know-how," rather than school-learned algorithms. Lave (1988) found similar results with homemakers. However, little is known about the particular aspects of contextualized situations that facilitate problem performance. Likewise, we have little evidence that an approach that capitalizes on constructive experiences with contextualized experiences would result in learning that readily transfers to abstract and/or novel contextualized situations.

Method

Participants were 142 sixth, seventh, and eighth grade students from a middle to high SES public school where integer operations are typically introduced during the spring of seventh grade. A "difficulty factors assessment" was designed and administered to all students. Sixteen forms counterbalanced problems varying with respect to 1) problem presentation (story vs. number-sentence), 2) number of arithmetic operations (one vs. two), 3) position of the unknown (start vs. result), and 4) for number-sentence problems only,

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representation of the unknown (box vs. blank). (An annotated example of items from one form is included.) Performances were analyzed to examine developmental trends relating to problem difficulties.

Sixth graders then participated in a one hour per week, 4 to 5 week simulation analysis and simulation building project. Initially, student pairs analyzed four teacher-researcher developed computer simulations (created with MicroWorlds Project Builder, LCSI). The first three modeled integer addition, subtraction, and multiplication, respectively, while the fourth modeled a combination of these. They then created their own simulations modeling the operation of their choice. (A complete description of the instructional intervention will be available in the full paper. Teacher-researcher created simulations and example student-created simulations will be demonstrated at the session.) Following the intervention period, each sixth grade participant was again administered the difficulty factors assessment. Pre- and post-test performances were analyzed to assess the effectiveness of the intervention for developing skill in using negative numbers in and out of context.

Results

Initial analyses of the pre-test difficulty factors assessment showed main effects for the position of the unknown and the representation of the unknown (number-sentence problems only). Not surprisingly, result-unknown problems were significantly easier than start-unknown problems regardless of other problem characteristics and regardless of grade level. More surprisingly, was a difference due to a subtle change in the way the unknown in a number sentence was represented. Students performed significantly better when the unknown was represented with a blank rather than a box regardless of other problem characteristics and regardless of grade level.

Interactions with grade level were found in a 3x2x2 (grade level by number of operations by problem presentation) analysis of variance with repeated measures on number of operations and problem presentation was computed. (Means and standard deviations reported in the full paper.) The significant main effects for grade level and number of operations indicated that students improve in negative number skill as they progress through their middle school years and that one-operation problems are substantially easier than two-operation problems. The problem presentation by grade level interaction effect ($F(2, 139)=8.67$, $p<.01$) indicated that 6th grade learners perform better on story problems than number-sentence problems, but that this performance difference diminishes with age/schooling/experience. More interestingly, the problem presentation by number of operations interaction effect ($F(1,139)=11.132$, $p<.01$) indicated that a story problem is only easier than its decontextualized counterpart when the problems involve one operation. The effect reversed for two operation problems, number-sentence problems with two operators were easier than story problems with two operators. In general we observed the following progression from easiest to the most difficult problems:

# of operators	Problem presentation	Unknown position	% correct
1	story	result	79%
1	story	start	74%
1	number-sentence	result	70%
1	number-sentence	start	66%
2	number-sentence	result	62%
2	number-sentence	start	58%
2	story	result	57%
2	story	start	46%

To assess the benefits of the instructional intervention for developing sixth graders' skills in dealing with integer operations, a 2x2x2 (math achievement level by problem presentation by time) analysis of variance with repeated measures on problem presentation and time was computed. The main effect for math achievement level (high vs. average/low) ($F(1,41)=22.5$, $p<.01$) revealed that high achieving math students performed better overall than average/low achieving students. The insignificant effects for time and time by math achievement level, coupled with the significant time by problem presentation and time by problem presentation by math achievement level interaction effects indicated that the intervention was associated with gains on story problems only. Furthermore, these gains were experienced by high achieving math students only.

Discussion

These results can be used to reflect on the relative benefits of 1) "situated" problem solving, in which problem solvers represent and perform calculations using specific features of the quantities and relations in the problem context, and 2) "symbolic" problem solving, in which problem solvers represent and perform calculations using a symbolic expression that abstracts away specific features of the context. Situated problem solving has the advantage of access to semantic information that constrains solution procedures (e.g., students are less likely to get 68.61 when adding 68.36 and 25 within the context of dollars and cents). Symbolic problem solving has the complementary advantage of stripping away potentially distracting context to create a "lightweight" representation that reduces working memory demands during solution performance.

The results of the difficulty factors assessment are consistent with the hypothesis that the advantages of situated strategies will be apparent on lower complexity problems (e.g., one operator problems) when there is little need for abstraction, whereas the advantages of a symbolic strategy will appear greater as the problem complexity increases (e.g., two operator problems). In particular, this hypothesis is consistent with the observed interaction between number of operators and problem presentation. Students' lower error rate on one-operator story problems relative to one-operator number sentences may result from differential use of situational constraints (provided by stories but not number sentences) to reduce arithmetic errors on story problems. For example, a problem involving one temperature change (say an increase of 5 degrees Celsius from a starting point of -3) readily alerts a solver to an incorrect procedure because the answer (say -8) doesn't make sense. In contrast, the greater complexity of two operator problems may have hindered direct use of a situated strategy and encouraged symbolic abstraction prior to arithmetic performance. Greater use of the symbolic strategy predicts better performance on the already-abstracted number sentences as such sentences are essentially a subgoal in applying the symbolic strategy to story problems. That is, if story problems are first translated to number sentences (or something like them), they must be more difficult to solve than the number sentences by themselves. These conclusions are tentative as more detailed analysis and coding of student errors and strategies, as can be identified from their shown work, may confirm or disconfirm specific hypotheses.

The results of the instructional intervention indicated positive learning gains as a possible consequence of the relatively short (5 hours) simulation analysis and simulation building project. These gains were limited, however, to story problems and to the best students. At this point we have no hypothesis for the lack of effect on the lower achieving students. The effect on story problems only, however, is consistent with cognitive theory (e.g., Singley & Anderson, 1989) that predicts transfer will be limited to fairly specific overlaps between instruction and target performance, particularly, contextualized instruction will improve contextual problem solving (as we saw here) while decontextualized instruction will improve decontextualized problem solving (as we've seen in traditional classes). These results suggest caution in the implementation of mathematics reforms which sometimes have eliminated basic skill practice in the over-zealous pursuit of more "meaningful", "authentic", "real-world" problem solving (cf., the whole-language vs. phonetics approach in reading). The difficulty factors assessment provided evidence for the importance of the symbolic strategy for dealing with increasing complexity in problem solving. Thus, even if real-world problem solving is the primary objective, practice in basic skills is critical to achieving that objective. Greater emphasis on applications is a welcome change for many classrooms that have traditionally over-emphasized basic skills, however, finding the appropriate balance appears to be right (and most difficult) approach.

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Items from Form 4a with annotations indicating condition.

[Story, two-operation, start-unknown, borrow situation.]
1 For his 8th grade entrepreneurship project Marco plans to sell loaves of homemade banana bread. He borrows 30 dollars to cover expenses. He sells loaves for 2 dollars each. How many loaves must he sell in order to show a profit of 4 dollars?

[Story, two-operation, result-unknown, even split situation.]
2 Lisa and Jena decide to evenly split the total of whatever they both win or lose at a game of chance. If Lisa won 4 dollars and Jena lost 10 dollars, how much do they each get in the split (use a negative number if they owe money)?

[Story, one-operation, start-unknown, temperature situation.]
3 A freezer's temperature starts at -2 degrees Celsius and cools at a steady rate. How many degrees has it cooled when it to reaches -14 degrees Celsius?

[Story, one-operation, result-unknown, climber situation.]
4 A rope climber climbs up 16 yards then slips down 21 yards. Where did she end up in relation to where she started?

[The unknown representation factor, blank vs. box, was specific to number-sentence problems and was a between-subjects factor. Thus, this form contains only blanks. A parallel form (4b) it had boxes in place of the blanks, but was otherwise identical.]
[Number-sentence, two-operation, start-unknown, even split analog.]

5 $(2 + \_\_) \div 2 = -6$

[Number-sentence, one-operation, result-unknown, borrow analog.]
6 $-20 + (3 \times 9) = \_\_$

[Number-sentence, one-operation, start-unknown, climber analog.]
7 $18 + \_\_ = -4$

[Number-sentence, one-operation, result-unknown, temperature analog.]
8 $-5 - 6 = \_\_$

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