Seeing Language Learning inside the Math: Cognitive Analysis Yields Transfer

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Abstract

Achieving and understanding effective transfer of learning requires a careful analysis of the hidden knowledge and skills to be transferred. We present an experiment that tests a subtle prediction of such an analysis. It concluded that a critical difficulty in students' learning to translate algebra story problems into symbolic expressions is in learning the grammar of such expressions. We hypothesized that exercises requiring students to substitute one algebraic expression into another would enhance students' algebraic grammar knowledge. This hypothesis led to a counter-intuitive prediction that learning to symbolize story problems could be better enhanced through practice on dissimilar looking substitution exercises than through practice on more similar looking story problems. We report on an experimental comparison involving 303 middle school students that supports this prediction. We discuss how having learners externalize a uniform abstract form and get interactive feedback on it may be important factors in enhancing transfer.

Keywords: cognitive task analysis; transfer; grammar learning; mathematics education.

Introduction

Humans learn language before they have a language to use to learn. Might the learning processes that make this amazing feat possible, like the capability to learn grammatical structures through experience without explicit instruction, be useful for other kinds of learning tasks? Once children have acquired language, are the cognitive functions employed in language learning no longer useful? For instance, as students take courses in complex academic topics, like algebra, does all that brain matter for language learning have nothing to do? Or is it possible that some of the same implicit learning mechanisms employed in language learning are useful for learning math and science?

This paper does not aim to provide conclusive answers to these questions, however, it does provide a compelling demonstration that grammar learning processes may be important in learning mathematics. Students may engage in such learning without explicit awareness and such implicit learning may be more prevalent in academic learning than is generally recognized (e.g., Alibali & Goldin-Meadow, 1993; Landay & Goldstone, 2007). In earlier work, we performed a cognitive task analysis of the important task domain of "symbolization", that is, the ability to model problem situations or "story problems" in algebraic symbols (Heffernan & Koedinger, 1997; 1998). Table 1 shows examples of symbolization problems, which ask students to translate a story problem into an algebraic expression. The obvious potential connection between language learning processes and this task is in learning to read and comprehend story problems. While such learning is indeed a significant challenge for elementary students (Cummins, Kintsch, Reusser, & Weimer, 1988), our past data provided evidence that comprehending story problems is no longer a major sticking point for most beginning algebra students.

This claim can be illustrated by an analogy to foreign language translation: Translating a story problem to algebra is like translating English to Greek. For an English speaker, the difficulty in translating to Greek is not comprehending the English, but generating the Greek. Similarly, the challenge for older students in a beginning algebra course is much less in understanding the English in which the story problems are written and more in being able to express that understanding algebraically, that is, in the language of algebra.

One indication that comprehension of algebra story problems is not a major sticking point for beginning algebra students comes from Heffernan and Koedinger's (1998) data showing that students can solve story problems (produce a value for the dependent or "y" variable when a value for the independent or "x" variable is given) much more accurately (63% correct) than they can symbolize (write an equation relating x and y) a story problem (18% correct). Since solving requires comprehension of the story, the performance difference is suggestive that symbolizing is problematic for students in ways beyond the demands of sentence comprehension. A second indication presents a contrast with a difficulty experienced by Artificial Intelligence systems programmed to solve story problems, namely that of understanding the arithmetic relationships between quantities described in the story (Bobrow, 1968). We created problems where natural implicit descriptions of such relationships (e.g., "Ms. Lindquist teaches 62 girls. Ms. Lindquist teaches b boys.") are supplemented (Heffernan & Koedinger, 1997) or replaced (Koedinger, Alibali, & Nathan, 2008) with explicit descriptions (e.g., "The number of students Ms. Lindquist teaches is equal to the number of boys plus the number of girls."), which are much easier for a program to process. We found, however, that providing such explicit descriptions does not

Table 1. Eight two-step symbolization items in order from easiest to hardest.

name	Item	Answer
cds	Mary opened a new music store. She got CDs delivered on her first day. She got 5 truck loads of CDs delivered. Each truck that arrived dropped off 12 boxes. Each box she received had ***c CDs***. Write an expression for how many CDs were delivered that first day.	5*12*c
mcdona	Mike starts a job at McDonald's that will pay him 5 dollars an hour. Mike gets dropped off by his parents at the start of his shift but he takes a taxi home that costs him 7 dollars. Mike works an ***h hour*** shift. After taking into account his taxi ride, write an expression for how much he makes in one night.	5*h-7
children	John and his wife Beth have been saving to give their 5 children presents for the holidays. John has saved 972 dollars for presents and Beth has saved ***b dollars***. They give each child the same amount. Write an expression for how much each child gets.	(972+b)/5
sisters	Sue made 72 dollars by washing cars to buy holiday presents. She decided to spend ***m dollars*** on a present for her mom and then use the remainder to buy presents for each of her 4 sisters. She will spend the same amount on each sister. Write an expression for how much she can spend on each sister.	(72-m)/4
students	Ms. Lindquist is a math teacher. Ms. Lindquist teaches 62 girls. Ms. Lindquist teaches ***f fewer boys*** than girls. Write an expression for how many students Ms. Lindquist teaches.	62+62-f
rowboat	Ann is in a rowboat on a lake. She is 800 yards from the dock. She then rows for ***m minutes*** back towards the dock. Ann rows at a speed of 40 yards per minute. Write an expression for Ann's distance from the dock.	800-40m
trip	Bob drove 550 miles from Boston to Pittsburgh to visit his grandmother. Normally this trip takes him ***h hours***, but on Tuesday there was little traffic and he saved 2 hours. Write an expression for what was his average driving speed.	550/(h-2)
jacket	Mark went to the store to buy jackets that cost ***d dollars***. When he got there the store was having a sale of 1/3 off the usual prices. Write an expression for how much the jacket cost him.	d-1/3*d

significantly improve the performance of beginning algebra students (77% on explicit vs. 79% on implicit in Koedinger, Alibali, & Nathan, 2008 and 53% vs. 50%, respectively, in Heffernan & Koedinger, 1997).

A third indication that problem comprehension is not a major sticking point identifies difficulties on the production side of the translation process (i.e., going from understanding to the target language, Algebra in this case) rather than the comprehension side (i.e., going from the source language, English story problems, to understanding). Heffernan and Koedinger (1997) contrasted the two-step problems shown in Table 1 (e.g., see the *students* problem in the fifth row) with matched one-step counter parts (e.g., see the first two rows in Table 2 for the one-step counterparts of the two-step *students* problem). In each matched set, the two one-step problems are designed to have essentially the same content as the two-step problem. Using the *students* problem as an example, the two-step problem requires the solver to understand that 1) the total number of Ms. Lindquist's students is the sum of the number of girls and number of boys and 2) that the number of boys is difference between the number of girls and the variable f. The one-step problem "a" in Table 2 requires understanding of first of these relationships and the other one-step problem "b" requires understanding the second of these. Heffernan and Koedinger (1997) found that student performance on symbolizing two-operator problems was significantly worse (40% correct) than combined performance on two matched one-operator problems (62% correct). (Note that average performance on a single one-operator problem is even better at 79% correct.)

The comprehension demands of the two one-operator problems are quite similar to that of the two-operator problem as the words and sentences used in each are substantially overlapping if not quite identical. The production demands, however, have an important difference. To correctly produce the algebraic expression for the one-step problems, 62+b and 62-f, learners need only acquire the mental equivalent of the grammar rule "expression => quantity operator quantity". However, this syntactic knowledge is not sufficient to produce two-operator symbolic expressions, like 62+62-f. To do so, requires the acquisition of knowledge equivalent to additional grammar rules that allow for an expression to be embedded inside another expression. More formally, producing two-operator symbolic expressions requires the equivalent of grammar rules like "expression => quantity operator expression" and "expression => expression operator quantity". Figure 1 illustrates how the first two of the three grammar rules above can combine to produce two-operator expressions like 62+62-f.

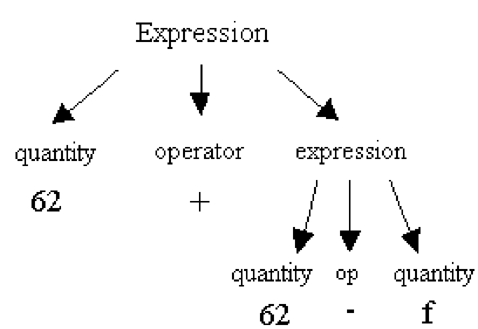

Figure 1: Grammar tree for a two-operator expression.

To be sure, we are not saying that students need to learn such grammar rules explicitly, but simply that they need to implicitly acquire the skills that are consistent with the patterns these rules describe. But the difference between two-step and one-step performance implicates such syntactic skill. In other words, that students are significantly worse at solving a single two-step problem than they are at solving both of the matched one-step problems is evidence that they lack implicit knowledge of grammar for combining expressions. There are alternative hypotheses to be sure (some of which were explored in Heffernan & Koedinger, 1997, 1998), but a strong test is to use this hypothesis to design purportedly better instruction and test whether it is indeed better.

So, for instruction, the ideal would be to find a task that isolates learning of these implicit "hidden" grammar rules. A task that does so is a substitution exercise, as illustrated in the last row of Table 2. This task requires students to produce of two operator expressions (and thus should exercise the hidden grammar rules) but without any of the requirements of comprehending a two-step story problem.

This leads us to a counter-intuitive hypothesis that instruction (substitution) that looks unlike the target objective (two-operator story problem symbolization) is going to lead to learning and transfer and, further, may do so better than instruction (one-operator story symbolization) that looks much more like the target objective. In particular, we hypothesize that practice on substitution exercises will transfer to better performance on translating algebra story problems into symbolic expressions. We will measure improvement by examining the differences on posttest two-step symbolization items between students who do substitution problems embedded within a problem set and students who only practice one-step symbolization problems within the problem set. As a pretest, both the treatment and control conditions begin with a measure of their ability to write one-step expressions before being presented with a two-step problem.

Method

The experiment was implemented inside the ASSISTment system and run in middle school classrooms in an urban school district outside of Boston, MA. The ASSISTment system is a web-based computer tutor authoring and delivery system designed to be used for both formative assessment and instruction (Razzaq et al., 2007). Instruction is provided by feedback on errors, on-demand hints, and scaffolding questions that reduce a problem into its components much like a simplified version of a Socratic dialogue.

Materials and Design

The materials for this study were the eight two-step story problems shown in Table 1 along with matched one-step and substitution items for each as illustrated in Table 2. This produces a pool of 32 items of which students saw 16 in one of two versions. Items were placed into the versions so that students never saw an item that has the same answer (or answer part) as another. The items were organized in three phases: 1) five pre-test items, 2) seven integrated instructed and post-test items, and 3) four filler items. The first two phases are relevant to the study design and are illustrated in Table 3. (The filler items are the one-step or substitution items the other condition received as instruction and were included to collect data on item difficulty.)

Table 2. The matched one-step problems (a & b) and substitution problem (c) for the two-step *student* item.

	Item	Answer
a	Ms. Lindquist is a math teacher. Ms Lindquist teaches 62 girls. Ms Lindquist teaches ***b boys***. Write an expression for how many students Ms. Lindquist teaches.	62+b
b	Ms. Lindquist is a math teacher. Ms Lindquist teaches 62 girls. Ms Lindquist teaches ***f fewer boys*** than girls. Write an expression for how many boys Ms. Lindquist teaches.	62-f
c	Substitute 62-f for b in 62+b Write the resulting expression.	62+62-f

In the pre-test phase, both groups did the same four one-step problems (labeled a or b in Table 3) depending on which version of the problem set received, followed by one two-step problem (labeled 0 in Table 3) depending on which version and order received. We created two "versions" to be evenly matched in difficulty by selecting two-step problems going down this list, cds, sisters, students, and jackets for version A and mcdonalds, children, rowboat, and trip for version B. Version A, then, had one-step and substitution items corresponding with cover stories mcdonalds, children, rowboat, and trip and vice versa for version B. We also created two orders of each version by reversing the sequence of the two-step problems, easy to hard (0, 1,2,3) vs. hard to easy (3,2,1,0). Thus, we expected order to have a significant effect on a pre-post comparison and controlled for it in the analyses below.

In the integrated instruction and post-test phase, students started with two instructional problems (either ab or cc in Table 3) and then alternated between two-step problems (1-3 in Table 3) and further instructional problems. As noted above, the four instructional problems come from the four base cover stories not used for the two-step problems, whether version A or B. The instructional problems corresponded with condition, one-step problems for the one-step condition and substitution problems for the substitution condition. For the one-steps, which come in a-b pairs (as illustrated in Table 2), two of type a and two of type b were selected from the four available cover story sets.

The two-step problems were ordered by difficulty based on a pilot study with students from the same grade and district and this order, from easiest to hardest, is shown in Table 1.

Table 3. Sequence of items for both conditions.

Condition	Pre-test	Instruct & test
One-step	abab0	ab1a2b3
Substitution	abab0	cc1c2c3

a & b = one-step, c = substitution, 0-3 = two-step

Given the nature of the ASSISTment system, all items are both assessment items (based on students' first response) and instructional items (based on feedback, hints, and scaffolding questions that may follow an incorrect response). The only difference between the two conditions is the placement of the substitution versus one-step items during the instruction.

Participants

The original data included 318 middle school students (N=158 one-step practice, N=160 substitution practice) using an on-line system during the 08-09 school year. The final data set included only those subjects who completed all 16 of the items in the problem set (four two-step, eight one-step, and four substitution) for a total of 303 participants (N=154 one-step, N=149 substitution).

Measures

The pre-test was designed to assess students' prior knowledge of translating story problem to algebraic expressions. It was the first five items in the item sequence and consisted of four one-step items and the first one two-step item. A pre-test measure was computed as the average of the two-step score and the average of the four one-step scores, thus appropriately giving more weight to the two-step item that is the goal of instruction. The posttest score was computed as the average of the scores on the last three two-step items. All pre and post-test scores were based on students' first attempt at an item such that either an incorrect entry or a hint request counted as an error.

Results

To test the main hypothesis that instruction on substitution tasks leads to better transfer of learning to two-step symbolization problems than does instruction on one-step symbolization problems, we performed an ANCOVA with pre-test as a covariate, condition and item order (easy-to-hard vs. hard-to-easy) as factors, and post-test as the dependent variable. As noted above, we included the order factor because of its obvious likely influence. We found significant effects of both factors, condition ($F(1,299) = 4.45$, $p < .05$) and order ($F(1,299) = 39.57$, $p < .001$), and of the pre-test covariate ($F(1,299) = 78.62$, $p < .001$). We found no other significant effects when we explored more complex models involving problem set version and the possible two- and three-way interactions with condition, version, and order.

Not surprisingly, high pre-tests are associated with higher post-tests and the easy-to-hard order yields lower post-test scores. With regards to condition, students in the substitution condition had similar pretest scores (M=.56) as students in the one-step condition (M=.57); however, the substitution group posttest scores (M=.39, SD=.35) were higher than the one-step group scores (M=.33, SD=.33). We used the ANCOVA results to compute adjusted posttest scores (M=.39 for substitution, M=.32 for one-step) and an effect size (Cohen's d = .29).

How Does Substitution Practice Help

To better understand how substitution practice may enhance learning of algebra symbolization skills, we investigated the errors students made on the posttest items. A common error category on two-step symbolization problems is to provide a 1-operator answer, for instance, "62-f" rather than "62+62-f". This error is consistent with a student whose only algebra grammar knowledge is "expression => quantity operator quantity". We hypothesized that substitution practice should aid the acquisition of grammar rules that allow for embedded expressions, like "expression => quantity operator expression". The addition of such knowledge should reduce the 1-operator responses to two-step problems.

We coded incorrect solutions in four error categories: 1-operator, 2-operator, missing parentheses, or hint/other. The most common error for both conditions is a 1-operator error. We found that the one-step group produces the 1-operator error (34%) somewhat more often than the substitution group (30%). This difference is larger for some problems and, in particular, appears to account for improved performance on four of the problems (cds, students, rowboat and trip) on which the one-step group is 9% worse than the substitution group (23% vs. 32%) and makes 12% (47% vs. 35%) more 1-operator errors. We found no consistent differences between conditions for 2-operator or hint/other errors. Three post-test problems require parentheses (sisters, children and trip) and on these, missing parentheses errors account for condition differences. The one-step group is 8% (34% vs. 42%) worse on these problems than the substitution group and makes 12% (25% vs. 13%) more missing parentheses errors.

We did not discuss parentheses in our brief characterization of the algebra grammar above, but the correct use of parentheses is clearly an important part of algebra expression structure. Consistent with the hypothesis that substitution practice should enhance algebra grammar learning, we indeed found a reduction in missing parenthesis errors in the substitution group relative to the one-step group.

One way grammar learning can be achieved is through the kind of implicit or non-verbal statistical learning mechanisms that are presumably used in first language acquisition. If these mechanisms are in part responsible for algebra grammar learning (see Li, Koedinger & Cohen, 2010 for a demonstration of the feasibility of such), then we might expect to see more frequent use of grammatical forms seen by those students who have seen such forms more frequently. Indeed, the one-step group sees 1-operator expressions more frequently and generates such expressions more frequently on post-test problems than the substitution group. In contrast, the substitution group sees more expressions with parentheses and generates such expressions more frequently on post-test problems than the one-step group.

In fact, these patterns appear not only in student errors, as discussed above, but also in their correct responses. On some two-step posttest problems (cds, students, and jackets) it is possible to produce a correct 1-operator solution (e.g., "60c" for 5*12c","124-f" for "62+62-f", 2/3 *d for d-1/3*d). The one-step group, despite doing generally worse on these problems (23% vs. 31%), actually produces twice as many correct 1-operator solutions as the substitution group (7.2% vs. 3.5%). It is also possible for students to produce correct answers that include parentheses on problems that do not require them (e.g., "62+(62-f)"). Again, consistent with the hypothesis that statistical properties of learning, like frequency, are operative even in formal domains like algebra, we find that the substitution group has more correct solutions that involve unnecessary parentheses than the one-step group (15% vs. 9.3%).

An astute reader may wonder about the following alternative interpretation of the observed overall differences in learning. Might the one-step group's experience generating 1-operator solutions simply be interfering with production of 2-operator solutions needed for correct performance on the two-step post-test problems? Or, to put it in more stark terms, might students in the substitution group simply be learning a shallow bias to generate 2-operator solutions and the one-step group students simply learning a shallow bias to generate 1-operator solutions? It is first worth emphasizing that, because of the instructional scaffolding for all on the two-step problems, neither group was exclusively seeing one response type or the other.

Certainly though, part of our hypothesis is that a shift in bias is causing improvement, but that that shift is in probabilities on implicit grammatical structure knowledge not in shallow or surface features. To be better, the substitution group students must not only avoid generating 1-operator solutions (note that they are not so easily biased that they stop making 1-operator errors), but also learn how to generate correct 2-operator solutions, including appropriate use of parentheses. If substitution group students were simply shallowly biased toward 2-operator solutions, we would expect them to perform worse on the four one-step problems they were given in the filler phase than the one-step group did on the same problems during instruction. In fact, both groups were 72% correct on one-step problems. Thus, the substitution group was not blindly over-generating 2-operator solutions.

Discussion and Conclusion

When we think about learning and transfer, it is tempting to think just in terms of the observable tasks between which transfer may occur. However, the vehicle of transfer is the knowledge the learner acquires from a source task and transfer occurs to the extent that that knowledge is relevant and employed in the target task (cf. Singley & Anderson, 1989). Careful cognitive task analysis regarding the underlying nature of the knowledge demands of tasks can thus provide insight into how best to achieve transfer. We presented an experiment that tested a subtle prediction of a prior data-driven cognitive task analysis. That analysis suggested that comprehending story problems tends not to be a major source of difficulty for students learning to translate story problems to algebra. Instead, learning to produce longer symbolic expressions is a more significant challenge and that students must acquire more sophisticated algebra grammar knowledge to meet this challenge. We hypothesized that practice on substitution tasks would assist students in extending their algebra grammar and, counter-intuitively, that such practice would yield better transfer to story problem symbolization than practice on simple story symbolization would. A classroom-based study with some 300 middle school students provided support for this hypothesis.

It may seem surprising that we found transfer from instruction on symbolization tasks, which have little natural language content, to story problem tasks and, even more, that such transfer is greater than from instruction on story problem tasks themselves (albeit simpler ones). After all, the literature and theory on analogical transfer (e.g., Gentner, 1983; Gick & Holyoak, 1983) suggests that people are particularly sensitive to surface features and have great trouble transferring experience from one situation (e.g., converging radiation treatment) to another with dissimilar surface features (e.g., converging military forces). How, then, does the instruction used in this study apparently help students acquire a relevant deep structure and transfer it from substitution tasks to surface-dissimilar story problem symbolization tasks?

An important observation here is that while these task categories (substitution and two-step story) do not have common surface features in their stimulus structure, they are similar in their response structure. The answer in both cases is a two-operator algebraic expression. To be sure, the correct responses to the instructional problems (analogical sources) and post-test problems (analogical targets) are not identical, nor even similar in surface characteristics -- for instance, "800-40x" has little or no surface similarity with "62+62-f". However, the structure of these responses, whether generated from a story problem or a substitution problem, is similar in underlying grammatical form ("expression => quantity operator expression").

Similarity in response structure is not enough to produce transfer. The well-known convergence tasks of Gick and Holyoak (1983) have an arguably similar response structure, yet learners show little transfer between such tasks under most instructional variations. What may be critical is that the learner externalizes the response, gets feedback and support to get the response right, and the external form is abstract and uniform (e.g., if a common converging arrow diagram was used in response to convergence tasks). In this study, the demands of both substitution and symbolization problems put the solution response into the world where it can be "re-perceived" (c.f., Goldstone, Landy, & Son, in press). Further, the kind of interactive instruction we employed (use of the ASSISTment tutor) guarantees that students get the response right before moving on. By generating, or at least perceiving a correct response, students may (implicitly) engage the same perceptually-grounded, similarity-based generalization processes on the response that they use on the task stimulus. Thus, they may develop better mental representations, whether grammar rules or "perceptual chunks" (Chase & Simon, 1973), of those response representations. Further, it may be important to the transfer result that the response representation is a uniform abstraction (i.e., algebraic expressions). This concise, unadorned representation may make it easier for pattern recognition mechanisms to learn the deep patterns (i.e., the algebraic grammar rules) needed for transfer (c.f., Kaminski, Sloutsky, & Heckler, 2008).

More practically, this research illustrates how a general instructional principle like starting simple (or mastery-based learning, Bloom, 1984) may not be effective if it is not accompanied with a careful cognitive task analysis of the target subject matter domain. Instruction that helps students master parts before helping them master the whole may seem obvious, however, what seem like "parts" on the surface may not be the right "cognitive parts" a learner needs to acquire. It is not particularly hard to identify the part-whole relationship between one-step and two-step story problems. Thus, the control condition in this study is not a straw man, but a reasonable application of part-task training principles and is representative of sequencing in math textbooks.

It is not *a priori* obvious, however, that substitution tasks are a "part" of two-step story problem symbolization. We came to that conclusion after a data-driven cognitive task analysis (cf., Clark, Feldon, van Merriënboer, Yates, & Early, 2007) that involved the analytic use of computational modeling (e.g., the grammar rule analysis). We believe that there is great promise for greatly improving the efficiency and effectiveness of instruction, even in well-investigated domains like algebra, through a combination of domain-general instructional principles and such detailed cognitive task analysis.

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